Preference for Flexibility and Dynamic Consistency

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Abstract

Dekel, Lipman, and Rustichini (2001) characterize preferences over menus of lotteries that can be represented by the use of a unique subjective state space and a prior. We investigate what would be the appropriate version of Dynamic Consistency in such a setup. The condition we find, which we call Flexibility Consistency, is linked to a comparative theory of preference for flexibility. When the subjective state space is finite, we show that Flexibility Consistency is equivalent to a subjective version of Dynamic Consistency and that it implies that the decision maker is a subjective state space bayesian updater. Later we characterize when a collection of signals can be interpreted as a partition of the subjective state space of the decision maker.

JEL Classification: D11, D81.
Keywords: Preference for Flexibility, Dynamic Consistency, Bayesian Updating, Subjective State Space.

1 Introduction

The issue of understanding how agents react to new information is a topic extensively studied in individual decision theory. Within the realm of the Savagean theory of decision making, where the state space is regarded as exogenously given, a fully rational individual is usually

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associated with the property of Dynamic Consistency. When, however, the decision model at hand regards the state space as endogenous, as in the recently developed dynamic theory of choice over menus, developing a theory of how agents react to information becomes a much less transparent problem. In a nutshell, this paper is an attempt to provide such a theory by searching for the natural version of Dynamic Consistency in that environment.

Consider the following situation. At 11 AM Jane has to choose a place for her evening drink with her friends. Suppose different places differ only in the menu of drinks that they offer and Jane always has only one drink. Let $X$ be the set of all possible drinks and $\mathcal{X}$ be the set of all conceivable drink menus, each menu being represented by a capital letter $A, B, C, \text{ etc.}$ As in Kreps (1979), we assume that Jane has a well-defined preference relation $\succeq$ on $\mathcal{X}$ and this relation exhibits preference for flexibility in the sense that, for any two menus $A$ and $B$,

$$A \succeq B \text{ implies } A \succ B.$$ 

Moreover, we assume that $\succeq$ admits the following representation:

$$A \succ B \text{ if } \sum_{s \in S} \pi(s) \max_{x \in A} U(x, s) > \sum_{s \in S} \pi(s) \max_{x \in B} U(x, s),$$

where $S$ is a finite state space, $\pi$ is a probability measure on $S$ and $U$ is a state-dependent utility function. The interpretation is that Jane is uncertain about the future and, in particular, she is not sure what kind of drink she will be in the mood for in the evening. The representation above thus says that she chooses a place that maximizes the expected utility she can get from the place’s drink menu, with respect to some prior $\pi$ about her future tastes.

At lunch, Jane will meet with her friends and one of them is going to be selected as the designated driver for the evening. If Jane gets to be the designated driver, and only in that situation, she very much appreciates if the place they go to has orange juice, as the other drinks do no matter for her. We represent the “Jane being the designated driver situation” by the state $s^* \in S$. If we use $j$ to represent the orange juice alternative, the discussion above can be formalized as:

$$U(j, s^*) > U(x, s^*) \text{ and } U(y, s^*) = U(x, s^*),$$

for all $x, y \in X$ distinct from $j$, and

$$U(j, s) < U(x, s),$$

for all $s \in S$.
for all \( x \in X \) and \( s \in S \), distinct from \( j \) and \( s^* \), respectively.

Suppose now that instead of choosing a place in the morning, Jane Örst goes for lunch with her friends where she is informed that she will not have to drive that evening. In terms of the representation above, this is equivalent to saying that she learns that the state \( s^* \) will not happen. In the present paper we wonder how in the current setup, where Jane’s state space is not observable, we can identify that this sort of situation is happening. More precisely, we investigate how we can learn from Jane’s choices that the signal she received was interpreted as information about her state space and that upon learning this information she acted in a dynamically consistent way. Because the state space \( S \) is not observable, we cannot simply say that Jane satisfies the standard Dynamic Consistency condition. The main goal of the present paper is to find a subjective version of this condition that can be applied to preference relations over a space of menus.

Without an exogenously specified state space, we cannot write a condition that explicitly deals with the fact that Jane receives new information at lunch. Nonetheless, Jane’s behavior still implies some consistency relating her preferences before and after lunch. Let \( \succeq \) represent her preference before lunch and \( \succeq^* \) the one after lunch. Now suppose that \( A \) and \( B \) are two menus such that \( A \succ^* B \), but \( B \succeq A \). That is, before lunch she considers menu \( B \) at least as good as menu \( A \), but after she learns that she will not have to drive in the evening, menu \( A \) becomes strictly more attractive than \( B \). We note that, by (1) and by the assumption about how Jane updates her preferences, this can happen only if \( j \in B \), but \( j \notin A \). Intuitively, the only difference between Jane’s preferences before and after lunch is that after lunch she no longer cares whether the place she goes has orange juice or not. So, if her before lunch preference relation values menu \( B \) more than her after lunch relation, it has to be because \( B \) offers exactly the alternative that loses its value once Jane learns she will not have to drive that evening.

Following the insight provided by this example, we investigate in this paper what would be the natural translation of Dynamic Consistency when the state space of the model is subjective. We work in the setup of Dekel et al. (2001)—henceforth DLR—and our main condition is a generalization of the idea discussed in the previous paragraph. In words, our condition says that if menus \( A \) and \( B \) are such that \( A \succ^* B \), but \( B \succeq A \), then it must be the case that \( \succeq \) sees some gain in flexibility when moving from menu \( A \) to menu \( A \cup B \) that \( \succeq^* \) does not see. Formally, it says that whenever \( A \succ^* B \) and \( B \succeq A \), there must exist a menu \( C \) such that \( A \cup B \cup C \sim^* A \cup C \), but \( A \cup B \cup C \succ A \cup C \).

The setup of lotteries of menus we use in this paper became popular after the works of DLR and Gul and Pesendorfer (2001). This two papers started a growing literature on
preferences over menus. Several papers in this literature, study variations of the self control representation in Gul and Pesendorfer (2001). (See Dekel, Lipman, and Rustichini (2009), Kopylov (2009b), Noor (2007), Noor and Takeoka (2010) and Stovall (2010).) Others derive particular cases of DLR’s additive representation that correspond to specific psychological phenomena. (See Barbos (2010), Dillenberger and Sadowski (2012a) and Sarver (2008), for example.)

Recently, a few papers in this literature have considered the problem of subjective acquisition of information. For example, Ergin and Sarver (2010) study an individual who is uncertain about her tastes, but can engage in costly contemplation before selecting an alternative from a menu. Their representation models contemplation strategies as subjective signals over a subjective state space. In a related paper, Ortoleva (2012b) models an individual who dislikes large choice sets because of the “cost of thinking” involved in choosing from them. In his most specialized representation, the individual thinks just enough to be able to make a choice.

Probably the two papers closest to this one, although still substantially different, are Dillenberger, Lleras, Sadowski, and Takeoka (2012) and Dillenberger and Sadowski (2012b). They work in a setup of menus of Anscombe-Aumann acts and model an individual who expects to receive a signal between the time of the choice of the menu and the time of the choice from the menu. The main differences between these two papers and the present one are that, first, in their case the individual expects to receive a signal after she has chosen a menu, but before she makes a choice from the menu. In our case, the signal arrives before the choice of the menu. Second, in their case the signal is subjective and appears only as part of the interpretation of the representation they derive. We work with objective signals, and what is subjective is how the individual interprets them.

Like this paper, Ozdenoren (2002) and Sadowski (2012) mix objective information with subjective states. Those two papers work with acts over an objective state space that return a different menu in each objective state of nature. The setup in this paper is slightly different, since we assume that each objective signal induces a different preference over menus. Of course there is a close relation between the two setups. In their case, it is as if the individual has to make a contingent plan for each possible objective signal she might receive.

In a setup with objective states, see Epstein and Le Breton (1993), Ghirardato (2002) and the references therein, for a discussion of Dynamic Consistency and Bayesian updating in the classic Savagen framework. For weakenings of Dynamic Consistency and a discussion of non-Bayesian updating rules, see Epstein (2006), Epstein, Noor, and Sandroni (2008) and Ortoleva (2012a). Finally, there is also some literature on updating in the context of the
multiple priors model of Gilboa and Schmeidler (1989) (see Epstein and Schneider (2003), Gilboa and Schmeidler (1993), Hanany and Klibanoff (2007) and Siniscalchi (2011)).

The remainder of our paper is organized as follows. We discuss the primitives of the model in Section 2. In particular, we introduce the concept of a finite Positive Additive Expected Utility (PAEU) representation, axiomatized by DLR, Dekel, Lipman, Rustichini, and Sarver (2007)—henceforth DLRS—and Dekel et al. (2009)—henceforth DLR2. In Section 3, we present a comparative theory of preference for flexibility and relate it to Dynamic Consistency and Bayesian updating. In particular, we define the fundamental notion of Flexibility Consistency in Section 3.1, and we present our main result, relating it to a subjective version of Dynamic Consistency in Section 3.2. The analysis in Sections 3.1 and 3.2 focuses on a single pair of relations and, therefore, concentrates on a unique signal. In Section 3.3, we extend the analysis to incorporate the possibility of multiple signals and, in particular, we characterize when a collection of signals forms a partition of the state space. In Section 4, we briefly discuss some fundamental aspects of the analysis in this paper. Section 5 concludes. The proofs of the main results as well as a discussion about the testability of the main postulate investigated in this paper appear in the appendix.

2 Preliminaries

We work within the setup of DLR. In what follows $X$ stands for a finite set of alternatives and $\Delta(X)$ for the set of probability measures on $X$. We view $\Delta(X)$ as a metric subspace of $\mathbb{R}^{[X]}$ and represent its elements by $p, q, r$, etc.. Let $\mathcal{X}$ represent the space of nonempty closed subsets of $\Delta(X)$. We write $int(\mathcal{X})$ to represent the subset of the elements of $\mathcal{X}$ that are included in the relative interior of $\Delta(X)$. That is, $int(\mathcal{X})$ is the set of all nonempty closed subsets of $\Delta(X)$ that include only lotteries with full support. The elements of $\mathcal{X}$ are represented by capital letters $A, B, C$, etc., and are called menus.

We consider binary relations $\succcurlyeq$ on $\mathcal{X}$. As usual, we denote the symmetric part of $\succcurlyeq$ by $\sim$ and the asymmetric part by $\succ$. We will work with the following definition:

**Definition 1.** We say that a relation $\succcurlyeq$ on $\mathcal{X}$ admits a finite Positive Additive Expected Utility (PAEU) representation if there exists a finite set $S$, a probability measure $\mu$ on $S$ and a function $U : S \times \Delta(X) \to \mathbb{R}$ such that

\[ U(s, \mu) = \sum_{x \in S} \mu(x) U(x, \mu) \]

for all $s \in S$ and $\mu \in \Delta(X)$.
1. For any two menus $A$ and $B$,

$$A \succ B \iff \sum_{s \in S} \mu(s) \max_{p \in A} U(s, p) \geq \sum_{s \in S} \mu(s) \max_{p \in B} U(s, p);$$

2. For each $s \in S$, there exists a nonconstant $u \in \mathbb{R}^X$ such that, for any $p \in \Delta(X)$,

$$U(s, p) = \sum_{x \in X} p(x) u(x);$$

3. $\text{Supp}(\mu) = S$ and, for each distinct $s$ and $s'$ in $S$, $U(s, \cdot)$ and $U(s', \cdot)$ are not positive affine transformations of each other.\(^2\)

When a relation $\succ$ admits a finite PAEU representation we call $\succ$ a finite PAEU preference. PAEU preferences without the restriction of a finite state space were axiomatized by DLR and DLRS. DLR2 introduced an axiom that characterized the finiteness of the state space $S$. (See also Kopylov (2009a) for a different finiteness axiom and a more detailed discussion about finite additive representations for preferences over menus.) The interpretation is that the agent solves a two stages decision problem. In the first stage the individual chooses a menu of options, knowing that in the second stage she will have to choose an option from that menu. This individual is uncertain about the future and, in particular, she is uncertain about what her tastes will be when she finally has to make a choice from a given menu—where each state of the world represents a different taste. She then chooses a menu in order to maximize her ex ante expected utility, taking into account that when the time arrives she will choose the best option from the menu, according to her taste at the time of choice.

In the representation above, the set $S$ is only an index set and it is not directly relevant. The relevant aspect is the set of ex post preferences induced by $\{U(s, \cdot) : s \in S\}$. Condition 2 in the definition above says that all these ex post preferences admit an expected-utility representation, while condition 3 requires that the index set $S$ contains no redundant states, in the sense that each state is associated with a different ex post preference, and no trivial states, in the sense that every state $s$ has positive probability. Following DLR, we refer to the set of expected-utility preferences induced by $\{U(s, \cdot) : s \in S\}$ as the subjective state space. DLR show that the subjective state space is unique, in the sense that any two finite PAEU representations of the same relation share the same subjective state space. Given condition 3, we might assume, without loss of generality, that the index set $S$ in a given finite PAEU

\(^2\)Notation: For a given probability measure $\mu$, we write $\text{supp}(\mu)$ to represent the support of $\mu$.  

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representation coincides with its subjective state space. From now on, whenever we write a state space $S$ it is understood that $S$ is the set of expected-utility preference relations on $\Delta(X)$ that corresponds to $\succsim$’s subjective state space. Also, for a given subjective state $s$, we write $\succeq_s$ to represent the expected-utility preference associated with $s$.

3 Observable Signals and Comparative Preference for Flexibility

One important feature of the preferences over menus literature is that the state space $S$ that appears in Definition 1, and even the subjective state space of a given PAEU preference, are not observable. The primitive of the model is simply a preference relation over the space of menus and the state space $S$ appears only in the representation of this relation. In what follows, we will investigate the situation in which there are observable signals that might affect the DM’s behavior, even though the link with the subjective state space is unknown to the outside observer. Intuitively, as in the example in the introduction, in a world with subjective states often the problem is not that we cannot observe when the agent receives a signal—we often know that the individual has received a given signal. At the same time, we have no means to understand how the agent interprets the signal in terms of her subjective state space. This is the key difficulty of our analysis, which sets it apart from the standard Savagean setup where signals are usually indexed by some event and are interpreted as if the agent learned that the associated event would occur for sure.

Technically, we will work with two different finite PAEU preferences. We will use the symbol $\succsim$ to represent the agent’s preferences before she receives any signal and we will use $\succsim^*$ to represent the agent’s preferences after she receives a specific signal. Of course, there is no reason for us to expect that there is only one possible signal: in principle, we could have a collection of $\succsim^*$’s, one for each possible signal. For simplicity, we focus on a single pair of relations for now, but the analysis can be easily generalized to incorporate more signals. (See Section 3.3 below for the details.)

3.1 Comparative Preference for Flexibility

As we will discuss, our notion of Subjective Dynamic Consistency is connected to standard notions of preference for flexibility. We begin with the following definition, due to DLR.

Definition 2. We say that a binary relation $\succsim$ on $\mathcal{X}$ values flexibility more than some
other binary relation \( \preceq^* \) on \( X \) if, for any two menus \( A \) and \( B \) with \( B \subseteq A \),

\[
A \succ^* B \text{ implies } A \succ B.
\]

In words, \( \preceq \) values flexibility more than \( \preceq^* \) if in any situation where \( \preceq^* \) strictly prefers the flexibility of having more options, so does \( \preceq \). When \( \preceq \) and \( \preceq^* \) are finite PAEU preferences, DLR, Theorem 2, shows the following.

**Lemma 1.** Suppose \( \preceq \) and \( \preceq^* \) are finite PAEU preferences, then \( \preceq \) values flexibility more than \( \preceq^* \) if and only if the subjective state space that represents \( \preceq \) is larger (in the inclusion sense) than the subjective state space that represents \( \preceq^* \).

In words, this result says that \( \preceq \) values flexibility more than \( \preceq^* \) if and only if every future subjective state considered possible by \( \preceq^* \) is also considered possible by \( \preceq \).

Lemma 1 characterizes the case in which \( \preceq \) values flexibility more than \( \preceq^* \). However, that definition is valid even if the differences between \( \preceq \) and \( \preceq^* \) go far beyond the way they value flexibility. For example, \( \preceq \) and \( \preceq^* \) may disagree on the comparison between two menus \( A \) and \( B \) and the reason for that be completely unrelated to the fact that \( \preceq \) values flexibility more than \( \preceq^* \). In some cases, it might be useful to be able to say that the only difference between \( \preceq \) and \( \preceq^* \) is the fact that \( \preceq \) values flexibility more than \( \preceq^* \). That is, we wish to capture the idea that any disagreement between \( \preceq \) and \( \preceq^* \) be only a consequence of \( \preceq^* \)'s higher desire for flexibility. This is the goal of the following property.

**Flexibility Consistency.** For any menu \( A \in X \) and menu \( B \in \text{int}(X) \), \( A \succ^* B \) and \( B \succeq A \) imply that there exists a menu \( C \) such that \( A \cup B \cup C \sim^* A \cup C \), but \( A \cup B \cup C \not\succ A \cup C \).

Intuitively, if \( \preceq \) and \( \preceq^* \) satisfy Flexibility Consistency and we have \( A \succ^* B \), but \( B \succeq A \), it must be the case that there exists at least one situation where \( \preceq^* \) sees no value in adding the options in \( B \) to \( A \), but \( \preceq \) still sees that as a strict improvement. This means that, whenever the two preferences disagree in the way described above, we can blame the disagreement on the fact that \( \preceq \) sees more value in the flexibility achieved by adding \( B \) to \( A \) than \( \preceq^* \).

Notice that in the statement of Flexibility Consistency we require that the menu \( B \) belong to \( \text{int}(X) \), as opposed to \( X \), which makes the axiom weaker. We will elaborate on that after the intuition for the proof of the main theorem, but for now it suffices to say that it is

\[^{3}\text{To be precise, DLR prove a more general version of Lemma 1, but the version stated here will be enough for our purposes.}\]

\[^{4}\text{This postulate makes use of an existential quantifier and, consequently, it is in principle not testable. In Section 4.3 we discuss a new postulate that is equivalent to Flexibility Consistency and is testable.}\]
possible that \( \succeq \) and \( \succeq^\ast \) differ only in the way they value flexibility, but still the property above is not satisfied by a pair of menus \( A \) and \( B \) if the menu \( B \) contains lotteries in the boundary of the simplex. Intuitively, the menu \( C \) that does the job “leaves the simplex”.

In the next lemma we show that if \( \succeq \) and \( \succeq^\ast \) are finite PAEU preferences and they satisfy Flexibility Consistency, then \( \succeq \) values flexibility more than \( \succeq^\ast \). Intuitively, if \( B \subseteq A \), then neither \( \succeq \) nor \( \succeq^\ast \) see any gain in flexibility when \( B \) is added to \( A \). Therefore, we will never be able to find a situation where \( \succeq \) sees a strict gain in adding the options in \( B \) to \( A \) and \( \succeq^\ast \) does not. But then, Flexibility Consistency implies that we must have \( A \succ B \) if \( A \succ^\ast B \).

We summarize this discussion with the following observation:

**Lemma 2.** Let \( \succeq \) and \( \succeq^\ast \) be two finite PAEU preferences. If \( \succeq \) and \( \succeq^\ast \) satisfy Flexibility Consistency, then \( \succeq \) values flexibility more than \( \succeq^\ast \).

**Proof.** Suppose \( B \subseteq A \) and \( A \succ B \). We can assume, without loss of generality, that \( A \) and \( B \) belong to \( \text{int} \, (\mathcal{X}) \).\(^5\) Now note that, for any menu \( C \), \( A \cup B \cup C = A \cup C \), so it is not possible to find a menu \( C \) with \( A \cup B \cup C \succ A \cup C \). By Flexibility Consistency, we must have \( A \succ B \).

In the next section we will see that the Flexibility Consistency property is closely related to the concept of Dynamic Consistency, well known in models where the state space is objective.

### 3.2 Subjective Dynamic Consistency

In the previous section we introduced the property of Flexibility Consistency. That property captures the idea of a relation differing from \( \succeq \) only because of its weaker preference for flexibility. We now show that when \( \succeq \) has a finite PAEU representation, Flexibility Consistency is equivalent to a subjective version of Dynamic Consistency and, consequently, has similar implications as this more standard property.

We are ready to state the main result of the paper.

**Theorem 1.** Let \( \succeq \) and \( \succeq^\ast \) be finite PAEU preferences. The following statements are equivalent:

1. \( \succeq \) and \( \succeq^\ast \) satisfy Flexibility Consistency;

\(^5\)If this is not the case just combine both of them with some lottery with full support and use Independence to finish the proof.
2. Let $S$ and $S^*$ be the unique subjective state spaces of $\succ$ and $\succ^*$, respectively, and let $U : S \cup S^* \to \mathbb{R}$ be such that, for any $s \in S \cup S^*$, $U(s,.)$ is an expected-utility function that represents $\succ_s$. For any two menus $A$ and $B$ with

$$\max_{p \in A} U(s, p) = \max_{p \in B} U(s, p) \text{ for all } s \in S \setminus S^*,$$

$$A \succ B \iff A \succ^* B;$$

3. For every finite PAEU representation $(S, \mu, U)$ of $\succ$, there exists $T \subseteq S$ such that $(T, \mu_T, U)$ represents $\succ^*$, where $\mu_T$ is the Bayesian update of $\mu$ after the observation of $T$.

Intuition for the proof. Given DLR’s uniqueness result, it is clear that the third statement implies the second. Conversely, the space of menus is rich enough for the second statement to imply the third, similarly to the way Dynamic Consistency implies Bayesian Updating in a world with objective states. We give the intuition for the equivalence of the first and third statements.

Suppose first that the third statement is true. This implies that, for any two menus $A$ and $B$,

$$\sum_{s \in T} \mu_T(s) \max_{p \in A} U(s, p) \geq \sum_{s \in T} \mu_T(s) \max_{p \in B} U(s, p) \iff \sum_{s \in T} \mu(s) \max_{p \in A} U(s, p) \geq \sum_{s \in T} \mu(s) \max_{p \in B} U(s, p).$$

So, if $A \succ^* B$ and $B \succ A$ for some pair of menus $A$ and $B$, it must be the case that there exists $s^* \in S \setminus T$ such that

$$\max_{p \in B} U(s^*, p) > \max_{p \in A} U(s^*, p).$$

Now, for each $s \in T$ with $\max_{p \in B} U(s, p) > \max_{p \in A} U(s, p)$, let $p^B_s$ be a lottery in $B$ that maximizes $U(s, p)$ in $B$. If $U(s^*, p^B_s) < \max_{p \in B} U(s^*, p)$, we can simply add $p^B_s$ to the menu $C$ we are going to use in the statement of Flexibility Consistency (see Figure 1A). If it turns out that $p^B_s$ also maximizes $U(s^*, .)$ in $B$, then we can simply choose a new lottery $q_s$ that lies in the same indifference curve of $p^B_s$, with respect to $U(s, .)$, that is, $U(s, p^B_s) = U(s, q_s)$, and that lies below the indifference curve of $p^B_s$ with respect to

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*Here we are implicitly requiring that $\mu(T) > 0$, so that $\mu_T$ is well-defined.*
$U(s^*, \cdot)$, that is $U(s^*, q_s) < U(s^*, p^B_s) = \max_{p \in B} U(s^*, p)$ (see Figure 1B). Notice that it is at this point that the restriction that $B$ belongs to $\text{int}(\mathcal{X})$ becomes important. Without this restriction we cannot guarantee that such a lottery $q_s$ exists (see Figure 1C). It is clear that for the menu $C$ constructed above we have $A \cup B \cup C \succeq^* A \cup C$, but $A \cup B \cup C \succ A \cup C$.

Conversely, suppose that $\succsim^*$ is not a Bayesian update of the representation of $\succsim$. This implies that there exists a representation $(S^*, \mu^*, U)$ of $\succsim^*$ such that $\mu^*$ is not the Bayesian update of $\mu$ after the observation of $S^*$. In Appendix A we show that in this case we can always find two menus $A$ and $B$ such that

$$\sum_{s \in S^*} \mu^*(s) \max_{p \in A} U(s, p) > \sum_{s \in S^*} \mu^*(s) \max_{p \in B} U(s, p),$$

but

$$\sum_{s \in S} \mu(s) \max_{p \in A} U(s, p) \leq \sum_{s \in S} \mu(s) \max_{p \in B} U(s, p)$$

and

$$\max_{p \in A} U(s, p) \geq \max_{p \in B} U(s, p) \text{ for all } s \in S \setminus S^*.$$  

In this case it is clear that, for any menu $C$ such that $A \cup B \cup C \succeq^* A \cup C$ we also have $A \cup B \cup C \succ A \cup C$. Since the conditions above imply that $A \succ^* B$ and $B \succeq A$, this contradicts Flexibility Consistency.

In the intuition for the proof of Theorem 1 we have explained why we have assumed that $B \in \text{int}(\mathcal{X})$ (instead of $B \in \mathcal{X}$) in the definition of Flexibility Consistency. For completeness, we give now a concrete example of two relations $\succsim$ and $\succsim^*$ such that $\succsim^*$ is a Bayesian update.
of \( \succ _ c \), but they do not satisfy the property in the definition of Flexibility Consistency for a pair of menus \( A \) and \( B \) with \( B \notin \text{int} (X) \):

**Example 1.** Let \( X := \{ x_1, x_2, x_3 \} \), \( S := \{ s_1, s_2, s_3 \} \) and suppose that \( u : S \times X \to \mathbb{R} \) is defined by \( u(s_1, x_1) = u(s_1, x_2) = u(s_2, x_1) = u(s_3, x_3) = 1 \) and \( u(s_1, x_3) = u(s_2, x_2) = u(s_3, x_1) = u(s_3, x_2) = 0 \). Finally, let \( \mu := (1/2, 1/6, 1/3) \) and \( \mu^* = (0, 1/3, 2/3) \).

Consider the relations \( \succ _ c \) and \( \succ _* \) that have PAEU representations \( (S, \mu, U) \) and \( (\{ s_2, s_3 \}, \mu^*, U) \), respectively, where, for each lottery \( p \in \Delta(X) \) and state \( s \in S \), \( U(s, p) := \sum_{x \in X} p(x) u(s, x) \).

Also, notice that \( \mu^* \) is the Bayesian update of \( \mu \) after the observation of \( \{ s_2, s_3 \} \). However, consider the menus \( A := \{ (0, 0, 1) \} \) and \( B := \{ (1, 0, 0) \} \). It can be checked that \( A \succ ^* B \) and \( B \succ A \). But notice that, for any menu \( C \), \( A \cup B \cup C \sim ^* A \cup C \) if and only if \( (1, 0, 0) \in C \). But then, for any such menu \( C \), \( A \cup B \cup C = A \cup C \) and it cannot be true that \( A \cup B \cup C \succ A \cup C \). That is, \( \succ _ c \) and \( \succ _* \) do not satisfy Flexibility Consistency.

### 3.3 Multiple Signals and Partitions

The analysis in the previous sections can be easily extended to the case of multiple signals. In fact, with multiple signals we can sometimes say something more about the information the agent expects to receive. Formally, we consider a finite collection of \( I + 1 \) finite PAEU relations \( \succ _ c \) and \( \{ \succ _{i}\} _{i \in I} \) such that, for each \( i \in I \), \( \succ _ c \) and \( \succ _{i} \) satisfy Flexibility Consistency. From Theorem 1 above, we know that each \( \succ _{i} \) can be represented as a Dynamically Consistent update of \( \succ _ c \) after the observation of some event \( T_i \). We impose two additional postulates relating \( \succ _ c \) and \( \{ \succ _{i}\} _{i \in I} \).

The first property we impose captures the idea that no subset of the relations in \( \{ \succ _{i}\} _{i \in I} \) exhausts all the flexibility seen by \( \succ _ c \). Formally, we impose the following postulate.

**No Flexibility Irrelevant Relation.** For every proper subset \( J \) of \( I \), there exist menus \( A \) and \( B \) in \( X \) such that \( B \subseteq A \), \( A \sim _i B \) for all \( i \in J \), but \( A \succ B \).

The next postulate imposes some consistency on how the relations in \( \{ \succ _{i}\} _{i \in I} \) and \( \succ _ c \) compare different menus. More specifically, it imposes that when all relations in \( \{ \succ _{i}\} _{i \in I} \) agree on the comparison of a pair of menus, \( \succ _ c \) also agrees with them.

**Unanimity Consistency.** For any two menus \( A \) and \( B \) in \( X \), if \( A \succ _{i} B \) for every \( i \in I \), then \( A \succ B \).

We can now state the following proposition.
Proposition 1. Let $I$ be a finite set and suppose that $\succsim$ and $\{\succsim_i\}_{i \in I}$ are finite PAEU preferences such that, for each $i \in I$, $\succsim$ and $\succsim_i$ satisfy Flexibility Consistency. The following statements are equivalent:

1. The collection $\{\succsim_i\}_{i \in I}$ and $\succsim$ satisfy No Flexibility Irrelevant Relation and Unanimity Consistency;

2. The collection $\{S_i\}_{i \in I}$ of the subjective state spaces used in the PAEU representations of the relations $\{\succsim_i\}_{i \in I}$ is a partition of $S$, the subjective state space used in the PAEU representation of $\succsim$.

4 Discussion

4.1 Subjective States and Measurable Signals

Theorem 1 characterizes a situation where (i) the DM receives a signal that she interprets as an event in her subjective state space, and (ii) upon learning this event she acts in a dynamically consistent way. In this sense, the analysis here parallels what is done in the objective state space case with a small difference. While in the objective state space case (i) is imposed at the outset, by identifying each possible signal with an event of the state space, in our case we characterize from choice when a given signal satisfies such a measurability condition.

With a subjective or objective state space, one can argue that concentrating on signals that are interpreted by the DM as events is somewhat restrictive, but we would like to point out that this restriction seems to be less severe in the subjective state space case. Recall that in the preferences over menus literature uncertainty takes the form of uncertainty about future tastes. Given objective and foreseen signals, it is natural that this signals, when relevant for the DM’s behavior, lead to new future tastes and, consequently, to new states. Consider our initial example. There, the possibility that Jane may turn out to be the designated driver induces the creation of a new state where orange juice is the favorite drink. If this possibility did not exist, such state would not exist either.

On the other hand, in the objective state space case, the states have physical meaning and what drives their creation is not the existence of foreseen signals, but the available bets in the situation at hand. For example, consider an Ellsberg paradox situation with a single urn with 50 black balls and a 100 balls that can be either red or blue. Here we have no
option but to define the state space as \{black, red and blue\}, since this is what is going to be revealed to the DM in the end. However, the natural signals here would be something like “there are at least 20 red balls among the 100 balls of unknown colors”, etc..

4.2 Unforeseen Signals and New States

This paper is about updating when the individual receives an objective and foreseen signal. One could think of a related situation where the individual receives an unforeseen signal that makes her aware of some new states of the world. Below we argue that we can also give an unforeseen contingencies interpretation to our main result.

Recently, Karni and Viero (2012) introduced what they call reverse Bayesianism rules. They basically ask the following question: suppose an individual is a subjective expected-utility maximizer over an objective state space \( S \) when she learns about some new and unforeseen states. How should the individual’s choices before and after she learns about the new states relate? The possibility they explore is that they are going to be related by a reverse Bayesianism rule. That is, her choices before she learns about the unforeseen states should be a Bayesian update of her choices after that.

Now suppose that we reverse the roles of the relations \( \succ \) and \( \succ^* \) in our analysis. That is, \( \succ^* \) is now the individual’s preferences before the signal and \( \succ \) is the individual’s preferences after she receives an unforeseen signal. Now the main result of this paper can be interpreted as a characterization of the situation in which the individual receives an unforeseen signal that makes her aware of some new states and her choices before and after the signal are linked by a reverse Bayesianism rule in the spirit of Karni and Viero (2012).

4.3 Testability

The Flexibility Consistency postulate makes use of an existential quantifier which makes it in principle not testable. However, we can equivalently state the axiom instead of for generic menus \( B \) and \( C \), for a menu \( B \) that belongs to a specific class of finite menus—which can be identified behaviorally—and for a menu \( C \) that is a subset of \( B \). This equivalent axiom is now testable. Since the identification of the mentioned class of finite menus is slightly technical, we relegate the details to section A.3 in the appendix.

Another question that concerns the testability of the model is the observability of the relations before and after the signal. The analysis in the paper is performed under the assumption that the individual expects to receive an objective signal and this situation is
repeated frequently. Recall the leading example of the paper. There, Jane goes for a drink with her friends every week. Sometimes she knows before she chooses the place if she will have to be the designated driver or not, sometimes she does not know. The important point is that these choices can be observed repeatedly. This situation is nowhere different from what happens in the objective state space case. In that case, it is also true that the analysis only makes sense if we can observe the individual’s choices before and after the signal repeatedly.

4.4 Infinite State Space

A natural question is whether a result similar to Theorem 1 can be obtained without the assumption of a finite subjective state space. The answer for that is a partial yes. On the one hand, it is indeed true that even with an infinite subjective state space Flexibility Consistency implies that the agent’s preferences satisfy a subjective version of Dynamic Consistency. On the other hand, it turns out that the space of menus is not rich enough in order to Dynamic Consistency to imply a Bayesian Updating result similar to the one in part 3 of Theorem 1.

Formally, even if $\succcurlyeq$ and $\succcurlyeq^*$ are not necessarily finite PAEU preferences, it is still true that statement 1 in Theorem 1 implies statement 2 in that same theorem. However, because the space of menus is not rich enough, it is no longer true that 2 implies 3. The infinite state space case is discussed in detail in Riella (2012).

5 Conclusion

We developed an updating theory in the world of preferences over menus. We worked in the framework of Dekel et al. (2001), where the state space is endogenously obtained as part of the representation of a preference relation over menus. We first searched for the appropriate adaptation of Dynamic Consistency for that environment and studied its consequences. We called this condition Flexibility Consistency, and we showed that when the subjective state space is finite it implies a Bayesian updating result in the preferences over menus world.

The most natural way to extend the analysis here would be to investigate the consequences the condition studied in this paper has for other models of preferences over menus. One such exercise is performed in de Moura and Riella (2012), where it is shown that Flexibility Consistency has similar implications when applied to the incomplete preferences version of DLR’s model axiomatized by Kochov (2007). Another candidate for a similar exercise is the menu preferences version of the maxmin model characterized by Epstein, Marinacci, and
Seo (2007).

A Appendix

A.1 Proof of Theorem 1

[1 \implies 2] Let \( S \) and \( S^* \) be the unique subjective state spaces of \( \succeq \) and \( \succeq^* \), respectively, and let \( U : S \cup S^* \rightarrow \mathbb{R} \) be such that, for any \( s \in S \cup S^* \), \( U(s, \cdot) \) is an expected-utility function that represents \( \succeq_x \). Now suppose that there exist two menus \( A \) and \( B \) with

\[
\max_{p \in A} U(s, p) = \max_{p \in B} U(s, p) \quad \text{for all } s \in S \setminus S^*,
\]

but either \( B \succeq A \) and \( A \succ^* B \) or \( B \succ A \) and \( A \sim^* B \). In the second case, pick any two menus \( D \) and \( E \) such that \( D \subseteq E \) and \( E \succ^* D \). Define \( \tilde{A} := \lambda A + (1 - \lambda) E \) and \( \tilde{B} := \lambda B + (1 - \lambda) D \) for \( \lambda \) small enough so that it is still true that \( \tilde{B} \succ \tilde{A} \). Note that, by construction, \( \tilde{A} \succ^* \tilde{B} \) and

\[
\max_{p \in \tilde{A}} U(s, p) \geq \max_{p \in \tilde{B}} U(s, p) \quad \text{for all } s \in S \setminus S^*.
\]

This shows that whenever the second statement is not satisfied, we can find menus \( A \) and \( B \) such that \( B \succeq A \), \( A \succ^* B \) and

\[
\max_{p \in A} U(s, p) \geq \max_{p \in B} U(s, p) \quad \text{for all } s \in S \setminus S^*.
\]

Now suppose \( C \) is a menu such that \( A \cup B \cup C \sim^* A \cup C \). From the representations of \( \succeq \) and \( \succeq^* \) it is clear that we must have

\[
\max_{p \in A \cup B \cup C} U(s, p) = \max_{p \in A \cup C} U(s, p) \quad \text{for all } s \in S^*.
\]

But it is also clear that

\[
\max_{p \in A \cup B \cup C} U(s, p) = \max_{p \in A \cup C} U(s, p) \quad \text{for all } s \in S \setminus S^*,
\]

which implies that \( A \cup B \cup C \sim A \cup C \). This contradicts the first statement. We conclude that \( 1 \implies 2 \).

[2 \implies 3] Let \( S \) and \( S^* \) be the unique subjective state spaces of \( \succeq \) and \( \succeq^* \), respectively. Consider any finite PAEU representation \( \langle S, \mu, U \rangle \) of \( \succeq \). Without loss of generality we may assume that \( U \) is defined over \( S \cup S^* \). It is not hard to see that the fact that \( \succeq^* \) is a finite PAEU preference implies that there exists a probability measure \( \mu^* \) over \( S^* \) such that \( \langle S^*, \mu^*, U \rangle \) represents \( \succeq^* \). Suppose \( \mu^* \) is not the Bayesian update of \( \mu \) after the observation of \( S^* \), which we will denote by \( \mu_{S^*} \), whenever it is well-defined. Fix any sphere \( E \in \text{int}(\mathcal{X}) \). That is, \( E \in \text{int}(\mathcal{X}) \) can be written as \( E := \{ q \in \Delta(\mathcal{X}) : d(p^*, q) \leq \delta \} \), for some \( p^* \in \Delta(\mathcal{X}) \) and \( \delta > 0 \). We note that, since for each \( s \in S \cup S^* \) \( U(s, \cdot) \) is an expected-utility function, for every \( s \in S \cup S^* \) \( U(s, \cdot) \) has a unique maximizer in \( E \). Moreover, if \( s \neq s' \), then the maximizer of \( s \) is different from the maximizer of \( s' \). Suppose first that \( \mu(S^*) = 0 \). In this case, define \( A := E \) and \( B := \cup \{ \arg \max_{p \in E} U(s, p) : s \in S \setminus S^* \} \). We note that \( A \succ^* B \), \( B \sim A \) and

\[
\max_{p \in A} U(s, p) = \max_{p \in B} U(s, p) \quad \text{for all } s \in S \setminus S^*,
\]

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which contradicts 2. If \( \mu(S^*) > 0 \), then \( \mu_{S^*} \) is well-defined and there must exist states \( \tilde{s}, s' \in S^* \) such that \( \mu^*(\tilde{s}) > \mu_{S^*}(\tilde{s}) \) and \( \mu^*(s') < \mu_{S^*}(s') \). Again, let \( E \in \text{int}(\mathcal{X}) \) be a sphere of center \( p^* \), and for each \( \lambda \in [0, 1] \) define \( D_\lambda := \lambda(p^*) + (1 - \lambda) E \). Now define \( A := \{ \arg \max_{p \in E} U(s, p) : s \in (S \cup S^*) \setminus \{ s' \} \} \) and \( B := \{ \arg \max_{p \in E} U(s, p) : s \in (S \cup S^*) \} \). Now, if \( A \supseteq B \), let \( \hat{A} := A \) and \( \hat{B} := B \). Now, if \( A \sim \hat{B} \) and \( \max_{p \in A} U(s, p) = \max_{p \in B} U(s, p) \) for all \( s \in S \setminus S^* \). This again contradicts 2 and we conclude that \( 2 \implies 3 \).

[3 \implies 1] Let \( (S, \mu, U) \) and \( (T, \mu_T, U) \) be finite PAEU representations of \( \succsim \) and \( \succsim^* \), respectively, where \( \mu_T \) is the Bayesian update of \( \mu \) after the observation of \( T \). Now suppose that \( A \in \mathcal{X} \) and \( B \in \text{int}(\mathcal{X}) \) are such that \( A \succsim B \) and \( B \succsim A \). It is clear that this can happen only if there exists \( s^* \in S \setminus S^* \) such that

\[
\max_{p \in B} U(s^*, p) > \max_{p \in A} U(s^*, p).
\]

We now show that this implies that there exists a menu \( C \) such that

\[
\max_{p \in A \cup B \cup C} U(s, p) = \max_{p \in A \cup C} U(s, p), \quad \text{for every } s \in S^*, \tag{2}
\]

but

\[
\max_{p \in A \cup B \cup C} U(s^*, p) > \max_{p \in A \cup C} U(s^*, p). \tag{3}
\]

For each \( s \in S^* \), define \( q_s \) as follows: if \( \max_{p \in A} U(s, p) \geq \max_{p \in B} U(s, p) \), let \( q_s \) be any lottery in \( \arg \max_{p \in A} U(s, p) \). If \( \max_{p \in B} U(s, p) > \max_{p \in A} U(s, p) \) and there exists \( q \in \arg \max_{p \in B} U(s, p) \) with \( U(s^*, q) < \max_{p \in B} U(s^*, p) \), let \( q_s := q \). We are left with the case where \( \max_{p \in B} U(s, p) > \max_{p \in A} U(s, p) \), but \( U(s^*, q) = \max_{p \in B} U(s^*, p) \) for all \( q \in \arg \max_{p \in B} U(s, p) \). We first note that this implies that it cannot be the case that \( U(s, .) \) is cardinally equivalent to \(-U(s^*, .)\). If this was the case, we would necessarily have \( \max_{p \in B} U(s, p) < \max_{p \in A} U(s, p) \). This implies that there exist lotteries \( p \) and \( p' \) such that \( U(s, p) = U(s, p') \), but \( U(s^*, p) < U(s^*, p') \). Fix \( q \in \arg \max_{p \in B} U(s, p) \) and let \( \xi := q + p - p'. \) Note that, for every \( \lambda \in (0, 1) \), \( U(s, \lambda q + (1 - \lambda) \xi) = U(s, q) \), but \( U(s^*, \lambda q + (1 - \lambda) \xi) < U(s^*, q) = \max_{p \in B} U(s^*, p) \). Since \( B \in \text{int}(\mathcal{X}) \), we have \( \lambda q + (1 - \lambda) \xi \in \Delta(X) \) when \( \lambda \) is large enough. In this case, let \( q_s := \lambda q + (1 - \lambda) \xi \) for some \( \lambda \in (0, 1) \) such that \( \lambda q + (1 - \lambda) \xi \in \Delta(X) \). Now define \( C := \{ q_s : s \in S^* \} \). It is clear that (2) and (3) hold for such menu \( C \) and, consequently, \( A \cup B \cup C \sim^* A \cup C \), but \( A \cup B \cup C \succ A \cup C \). That is, \( \succsim \) and \( \succsim^* \) satisfy Flexibility Consistency.

### A.2 Proof of Proposition 1

Suppose the collection \( \{S_i\}_{i \in I} \) of the subjective state spaces used to represent the relations \( \{\succsim_i\}_{i \in I} \) is a partition of the subjective state space \( S \) used to represent \( \succsim \). Fix any representation \( (S, \mu, U) \) of \( \succsim \). That \( \{\succsim_i\}_{i \in I} \) and \( \succsim \) satisfy Unanimity Consistency is an easy consequence of Theorem 1.3. Now fix any \( \emptyset \not= J \subseteq I \) with \( J \neq I \), and let \( A \) be any sphere in \( \mathcal{X} \). For each \( s \in S \), let \( q_s \) be the unique maximizer of \( U(s, .) \) in \( A \). Now define \( B := \{ q_s : s \in \cup_{i \in J} S_i \} \). It is clear that \( A \sim_i B \) for every \( i \in J \), but \( A \succ B \). That is, \( \succsim \) and \( \{\succsim_i\}_{i \in I} \) satisfy No Flexibility Irrelevant Relation.

Conversely, suppose \( \succsim \) and \( \{\succsim_i\}_{i \in I} \) satisfy No Flexibility Irrelevant Relation and Unanimity Consistency. Let \( \{S_i\}_{i \in I} \) be the subjective state spaces used in the representations of the relations \( \{\succsim_i\}_{i \in I} \) and \( S \) the
one used in the representation of $\mathcal{Z}$. Fix any representation $(S, \mu, U)$ of $\mathcal{Z}$. By an argument very similar to the one we used in the sufficiency part of the proof, we can show that $\mathcal{Z}$ and $\{\mathcal{Z}_i\}_{i \in I}$ satisfy No Flexibility Irrelevant Relation only if $\cup_{i \in I} S_i \neq S$ for every $J \subseteq I$ with $J \neq I$. Let’s now show that for any distinct $i, j \in I$ we have $S_i \cap S_j = \emptyset$. For that, pick any sphere $E \in \mathcal{X}$ and let $p^*$ be its center. Suppose that there exist distinct $i, j \in I$ with $S_i \cap S_j \neq \emptyset$ and pick any state $\hat{s} \in S_i \cap S_j$. For each $\lambda \in [0, 1]$, let $p_{\lambda} := \lambda p^* + (1 - \lambda) q_i$, where $q_i$ is the unique maximizer of $U(\hat{s}, \cdot)$ in $E$. Now let $J := \{i \in I : \hat{s} \in S_i\}$. For each $i \in J$, pick $s_i \in S_i \setminus \cup_{j \in I \setminus \{i\}} S_j$. Now, for each such $s_i$, define $\lambda_{s_i} \in \mathbb{R}_{++}$ by

$$\lambda_{s_i} := \frac{\mu(\hat{s})(U(\hat{s}, q_i) - U(\hat{s}, p^*))}{\mu(s_i)(U(s_i, q_{s_i}) - U(s_i, p^*))}.$$ 

We note that, by choosing $\lambda$ small enough, we can guarantee that $\lambda_{s_i} \in (0, 1)$ for every $i \in J$ and that $U(s_i, \lambda_{s_i} p^* + (1 - \lambda_{s_i}) q_{s_i}) > U(s_i, q_{s_i})$, for every $s \in S$ distinct from $s_i$, where, for each $s \in S$, $q_s$ is the unique maximizer of $U(s_i, \cdot)$ in $E$. Now define menus $A$ and $B$ by

$$A := \{q_s : s \in S \setminus (\cup \{s_i : i \in J\})\} \cup \{\lambda_{s_i} p^* + (1 - \lambda_{s_i}) q_{s_i} : i \in J\}$$

and

$$B := \{\lambda p^* + (1 - \lambda) q_i \} \cup \{q_s : s \in S \setminus \{\hat{s}\}\}.$$ 

We note that, for every $i \in J$,

$$\sum_{s \in S_i} \mu(s) \left( \max_{p \in A} U(s, p) - \max_{p \in B} U(s, p) \right) = \mu(\hat{s}) \lambda(U(\hat{s}, q_i) - U(\hat{s}, p^*)) - \mu(s_i) \lambda_{s_i}(U(s_i, q_{s_i}) - U(s_i, p^*)) = 0,$$

while for every $i \in I \setminus J$ we have $\max_{p \in A} U(s, p) = \max_{p \in B} U(s, p)$ for every $s \in S_i$. Consequently, we have $A \sim_i B$ for every $i \in I$. Finally, note that

$$\sum_{s \in S} \mu(s) \left( \max_{p \in A} U(s, p) - \max_{p \in B} U(s, p) \right) = \mu(\hat{s}) \lambda(U(\hat{s}, q_i) - U(\hat{s}, p^*)) - \sum_{i \in J} \mu(s_i) \lambda_{s_i}(U(s_i, q_{s_i}) - U(s_i, p^*)),$$

while from

$$\mu(\hat{s}) \lambda(U(\hat{s}, q_i) - U(\hat{s}, p^*)) - \mu(s_i) \lambda_{s_i}(U(s_i, q_{s_i}) - U(s_i, p^*)) = 0$$

for every $i \in J$, we get

$$|J| \mu(\hat{s}) \lambda(U(\hat{s}, q_i) - U(\hat{s}, p^*)) - \sum_{i \in J} \mu(s_i) \lambda_{s_i}(U(s_i, q_{s_i}) - U(s_i, p^*)) = 0.$$

Since $U(\hat{s}, q_i) - U(\hat{s}, p^*) > 0$, this implies that

$$\sum_{s \in S} \mu(s) \left( \max_{p \in A} U(s, p) - \max_{p \in B} U(s, p) \right) < 0.$$ 

That is, $B \succ A$, which contradicts Unanimity Consistency. We conclude that for every distinct $i, j \in I$ we have $S_i \cap S_j = \emptyset$ and, consequently, $\{S_i\}_{i \in I}$ is a partition of $S$. 

\[\blacksquare\]
A.3 Flexibility Consistency and Testability

Given the assumption that each future state \( s \in S \) is associated with a different expected-utility preference represented by the function \( U(s,\cdot) \), if we pick a menu \( E \subseteq \Delta(X) \) which is a sphere (that is, there exists \( p \in \Delta(X) \) and \( \delta > 0 \) such that \( E = \{ q \in \Delta(X) : d(p,q) \leq \delta \} \)) we can be sure that each state \( s \in S, U(s,\cdot) \) is maximized by a different lottery \( p^E_s \in E \). In fact, every menu that can be written as a mixture of a generic menu and a sphere satisfies this property. Given our finiteness of the state space assumption, every menu is indifferent to a finite subset of itself, which makes it possible for us to identify a class of finite menus in which we are sure that, for every \( s \in S, U(s,\cdot) \) is maximized by a different lottery in these menus. Formally, we have the following definition:

**Definition 3.** We say that a finite menu \( A \) is a **finite menu with unique maximizers** if there exists a menu \( B \), a sphere \( E \subseteq \Delta(X) \) and \( \lambda \in [0,1) \) such that \( A \subseteq \lambda B + (1-\lambda) E \) and \( A \sim \lambda B + (1-\lambda) E \).

It turns out that a version of Flexibility Consistency that concentrates on finite menus with unique maximizers is enough to deliver the result in Theorem 1. Consider the following Postulate:

**Flexibility Consistency with Unique Maximizers.** For any menu \( A \in \mathcal{X} \) and finite menu \( B \) with unique maximizers, \( A \succ^* B \) and \( B \succ A \) implies that there exists a subset \( C \) of \( B \) such that \( A \cup B \cup C \sim^* A \cup C \), but \( A \cup B \cup C \succ A \cup C \).

If we replace Flexibility Consistency by the postulate above in Theorem 1 all the statements in that theorem remain true. The proof of this fact, which is an easy adaptation of the proof of Theorem 1 is left to the reader.

**References**


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