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A Simple Method of Elicitation of Preferences under Risk*

Patrícia Langsch Tecles† José Guilherme de Lara Resende‡

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Abstract

This study estimates the utility of lotteries and the degree of loss aversion of participants in an experiment, using prospect theory. For this purpose, we apply the parametric method proposed by Abdellaoui et al. (2008) to preferences observed in a computer-based experiment conducted at Universidade de Brasília. Most participants displayed risk aversion for gain prospects and risk propensity for loss prospects. Real incentives for loss prospects led to a greater concavity of the utility function than the one estimated by Abdellaoui et al. (2008). We observed reversals in behavior toward risk in the presence of a certain gain or loss in the prospect, which implies an overweight of the probability weighting function and is a new behavioral finding. Moreover, three different measures of loss aversion are discussed and, when applied to the experimental data, they were more appropriate with its theoretical definition than the most widely used measure of Tversky and Kahneman (1992).

Keywords: Prospect theory. Utility measurement. Loss aversion.

JEL classification: C91, D81, D01

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†Universidade de Brasília. E-mail: ptecles@gmail.com

‡Universidade de Brasília, campus universitário Darcy Ribeiro, caixa postal 04587, Brasília, DF. E-mail: jglresende@unb.br

1 Introduction

This study estimates the utility function and loss aversion of participants using data from an experiment and analyzes their decision making process of alternatives that involve risk. We apply the method developed by Abdellaoui et al. (2008) and make three contributions to their original work.

First, we use a system of monetary incentives that covers prospects[1] with gains and prospects with losses. Few studies provide real stimulus in the loss dimension, since it is not possible to impose losses of their own money to participants in experiments. Some studies circumvent this restriction by giving money in advance, which participants can lose partially or in its totality over the course of the experiment. Ganderton et al. (2000) investigate decisions about purchasing insurance using such a scheme. However, one should ensure that participants see the negative results as losses and not just as lower gains (Laury et al., 2009). Thaler and Johnson (1990) show that an initial outlay can turn people more prone to risk, a phenomenon called *house money effect*. In our study, in order to provide better incentives, a lottery over gains is drawn near the end of the experiment and the participant receives an amount according to the choice made. After that, a loss lottery is drawn and then the participant may lose some or all of this previous gain, in a similar fashion as Laury and Holt (2008) experiment on risk aversion. Thus, participants answer questions concerning losses without knowing the amount won, preventing that they realize their gain, what could induce a riskier behavior.

Second, participants of the experiment are not only graduate students, as in Abdellaoui et al. (2008), but also undergraduates in economics. We test if undergraduates have a lower understanding of probabilities and decision making processes. The aim is to compare results between the two groups and check if initial instructions and examples are sufficient to make it clear how the experiment works. This helps to extend experiments in the area of risk and uncertainty to wider audiences, bringing new information about choices, such as the comparison between individual characteristics.

Finally, we survey different forms of computing loss aversion, discussing when each one is more appropriate. Abdellaoui et al. (2008) use the definition adopted by Tversky and Kahneman (1992). As we discuss in section 2.1, this measure is satisfactory when the utility over outcomes displays the same curvature for gains and losses, i.e., when preferences involving losses are the mirror image of preferences on gains. Kahneman and Tversky (1979) call this trait of preferences the *reflection effect*. Most studies that estimate similar parameters for the utility of gains and losses use aggregate data. Estimates using individual data find find curvature parameters that greatly vary between gain and loss prospects (Abdellaoui et al., 2007; Gonzalez and Wu, 1999). Furthermore, Laury and Holt (2008) show that the reflection effect is considerably reduced when real incentives are used for choices involving losses. Therefore, in addition to employing the definition of Tversky and Kahneman (1992), we discuss three additional forms of measuring loss aversion, based on Kahneman and Tversky (1979), Tversky and Kahneman (1991) and Schmidt and Zank (2005).

Next section reviews the literature and the theoretical basis behind the procedures to estimate individual utility functions. Section 3 presents the model of utility elicitation based on Abdellaoui et al. (2008). We apply this model to the data obtained in an experiment conducted by us at Universidade de Brasília, described in Section 4. Section 5 discusses the results and Section 6 concludes.

[1]The words prospects, lotteries and gambles have the same meaning in this study.

2 Literature Discussion

2.1 Expected Utility Theory and its Shortcomings

Traditional consumer theory assumes that individuals maximize their utility. In a setup without uncertainty, the utility is a subjective value assigned to a given good or bundle of goods, where higher values are associated to preferred bundles. However, individual choices often involve uncertainty, such as the price of a product in the future, the return of a financial asset or the result of a bet. The concept of utility can be extended to risk situations where choices are made between prospects over a set of possible outcomes. A prospect with risk (a roulette lottery, in Anscombe and Aumann (1963)'s terminology) is characterized by its outcomes and their probabilities of occurring. Outcomes can include many things, such as goods and moneys.

Once one estimates an individual utility function and, therefore, the consumer's preferences, it is possible to model this person behavior. However, the utility cannot be directly observed. How then we measure the utility of lotteries and investigate how agents make their choices? Traditional theory is based on the idea of *expected utility*, given by the sum of the utilities of the possible outcomes, weighted linearly by their probabilities.

Expected Utility Theory (EUT) is the most famous theory used to model the decision making process under risk. Von Neumann and Morgenstern (1947, p. 26) formulated axioms necessary to the existence of a utility function that represents preferences over risk prospects and has an expected utility form. A utility function V of a prospect g has the expected utility property if for the outcomes $a_1, a_2, \ldots, a_n$, with respective utilities $u(a_1), u(a_2), \ldots, u(a_n)$ and respective probabilities $p_1, p_2, \ldots, p_n$, one has $V(g) = p_1u(a_1) + p_2u(a_2) + \cdots + p_nu(a_n)$. That is, the utility of a lottery is the sum of the utilities of its possible outcomes, weighted linearly by their corresponding probabilities. Lottery g is preferable to lottery f if and only if $V(g) \geq V(f)$.

In the following decades, EUT became dominant in economics, but several descriptive shortcomings were observed. Kahneman and Tversky (1979) gathered data from several experiments refuting the interpretation and application of EUT. Below are some examples of problems investigated. In each problem, people should choose between prospects involving a gain or a loss with some probability. The lotteries are represented by omitting the chance of a null result, no gain or loss. The values are given in pounds of Israel, as reported by Kahneman and Tversky (1979), and the percentage of participants who chose each alternative are in brackets.

Problem 1. Choose between:

A: 6,000 with probability 0.45 [14]	B: 3,000 with probability 0.90 [86]*

Problem 2. Choose between:

C: 6,000 with probability 0.001 [73]*	D: 3,000 with probability 0.002 [27]

Participants who prefer option B over option A and are modeled using expected utility theory (and, without loss of generality, assuming $u(0) = 0$), must satisfy the equation below:

$$0.90u(3,000) > 0.45u(6,000) \qquad \Rightarrow \qquad 0,002u(3.000) > 0,001u(6.000),$$

where we divided the first inequality by 450 to obtain the second, an algebraic procedure that does not violate the tenets of expected utility theory. Therefore, participants that chose B over A and choose C over D in problem 2 display a behavior that contradicts EUT. This behavior happens for a large percentage of people that participates in this experiment.

Thus, expected utility theory cannot explain the behavior of an individual who opts for both B and C, because these choices contradict the independence axiom, the bedrock of EUT. The same often occurs when there is an overvaluation of outcomes associated with the 100%-probability, that is, with certain outcomes. This effect, called *sure-thing effect*, can also lead to contradictions, as in the famous example based on Allais (1953). Kahneman and Tversky (1979) present this famous shortcoming in the following fashion.

Problem 3. Choose between:

A: 4,000 with probability 0.80 [20]	B: 3,000 with probability [80]*

Problem 4. Choose between:

C: 4,000 with probability 0.20 [65]*	D: 3,000 with probability 0.25 [35]

Most participants prefer B over A in problem 3 and C over D in problem 4. Under expected utility theory, the choice of B over A implies that:

$$u(3,000) > 0.8u(4,000) \qquad \Rightarrow \qquad 0.25u(3,000) > 0.2u(4,000).$$

An individual who prefers A to B must choose, according EUT's axioms, C over D. The above pattern of choices also violates the tenets of expected utility theory. In particular, the independence axiom is violated: if B is preferred to A, then B with the composition of any probability (B, p) must be preferable to the composition (A, p). Thus, the multiplication of the probabilities by 0.25 in the first choice should not reverse the preference, as it happens. This behavior is exhibited by a large percentage of participants and it is not a mere fluke in their responses. The utility of risky prospects does not appear to be linear in probabilities.

Kahneman and Tversky (1979) also disputed that individuals make decisions by taking in account their final states of wealth. Instead, they argued that choices are made by comparing the probabilities of gains and losses relative to a reference point. This reference point generally corresponds to the level of initial wealth, but may vary within the problem, according to the setup presented. Consider problems 5 and 6 below.

Problem 5. You receive 1,000 and have to choose between:

A: 1,000 with probability 0.50 [16]	B: 500 with certainty [84]*

Problem 6. You receive 2,000 and have to choose between:

C: -1,000 with probability 0.50 [69]*	D: -500 with certainty [31]

The majority of participants choose the second option in problem 5 and the first option in problem 6, though, in terms of final outcomes, both set of choices are identical:

A = 2,000 with probability 0.50 and 1,000 with probability 0,50 = C

B = 1,500 with certainty = D.

People seem to ignore the common components in these prospects and focus on the differences between them. Thus, problem 5 is seen as a lottery that involves gains and problem 6 is seen as a lottery that involves losses, and this difference in perception generates conflicting choices.

Kahneman and Tversky (1979) also challenged the assumption of pervasive risk aversion, i.e., that the expected value of a prospect is almost always preferable to the prospect. Friedman and Savage (1948) also discussed behavior that do not display risk aversion everywhere, arguing that a consumer can be risk averse for relatively high values and risk loving for relatively low values. Kahneman and Tversky (1979) found that the people are usually risk loving for losses, preferring the prospect to its expected value. They suggested a pattern of choices that became known in the literature as *the fourfold pattern of risk attitudes*, summarized in table 1. The fourfold pattern of risk attitudes describes a common type of behavior observed in experiments: risk aversion for gains with large probability and for losses with small probability and risk propensity for gains with small probability and for losses with large probability. This pattern is used, for example, to explain the purchase of lottery tickets and insurance by the same individual (Tversky and Kahneman, 1992; Harbaugh et al., 2010).

Table 1: *Fourfold pattern of risk attitudes*

Probability	Gains	Losses
Small	Risk Propensity	Risk Aversion
High	Risk Aversion	Risk Propensity

2.2 Prospect and Cumulative Prospect Theories

Using evidence collected in several experiments, Kahneman and Tversky (1979) proposed *prospect theory* (PT) to better describe the behavior of people in decisions involving risk. This theory brings two major changes to the expected utility framework: outcomes are viewed either as gains or losses from a reference point and the value of each outcome is multiplied by a decision weight given by a non-linear transformation of the prospect's probability.

Prospect theory applies to lotteries with at most two outcomes different from zero. In a later work, Tversky and Kahneman (1992) developed *cumulative prospect theory* (CPT) for lotteries with any number of results. Instead of using a transformation over probabilities, they assume a transformation over the cumulative distribution function[2]. The experiment that we conducted involves lotteries with only two possible outcomes. In this case, the original prospect theory and its cumulative version differ in only one respect: the latter allows different decision weights for gains and losses.

Let $(x, p\,; y)$ denote a lottery that gives outcome x with probability p and outcome y with probability $1 - p$. These outcomes represent changes in relation to a reference point, being viewed as gains or losses, not as the end position. A final wealth value of 100, for example, is seen as a gain of 5, when initial wealth is 95, and as a loss of 5, when initial wealth is 105. Utility V is determined by a

[2]This solves PT's problem of not complying with stochastic dominance. As shown by Gonzalez and Wu (2003), CPT has also implications on the psychological process of choices that differ from PT: equal probabilities may receive different weights according to the ranking of the results to which they are associated.

probability weighting function, denoted by w^+ for gains and by w^- for losses. The value function (or utility function over outcomes) is denoted by v. According to prospect theory, a lottery that involves only gains ($x \geq y \geq 0$, by convention) is evaluated by:

$$V(x, p\,; y) = v(y) + w^+(p)[v(x) - v(y)]\,,$$

and a prospect that involves only losses ($x \leq y \leq 0$) is evaluated by:

$$V(x, p\,; y) = v(y) + w^-(p)[v(x) - v(y)]\,.$$

The functions v, w^+ and w^- are assumed to be strictly increasing and to satisfy $v(0) = 0$, $w^+(0) = w^-(0) = 0$ and $w^+(1) = w^-(1) = 1$. The idea with the evaluation above is that there is at least a gain y (or a loss) with certainty, whatever the result is, and an additional gain (or loss) of $x - y$ with probability p. Equation 1 can be rewritten as:

$$V(x, p\,; y) = w^+(p)v(x) + [1 - w^+(p)]v(y)\,.$$

Finally, when the prospect has mixed results, $x > 0 > y$, the utility is given by:

$$V(x, p\,; y) = w^+(p)v(x) + w^-(1 - p)v(y)\,.$$

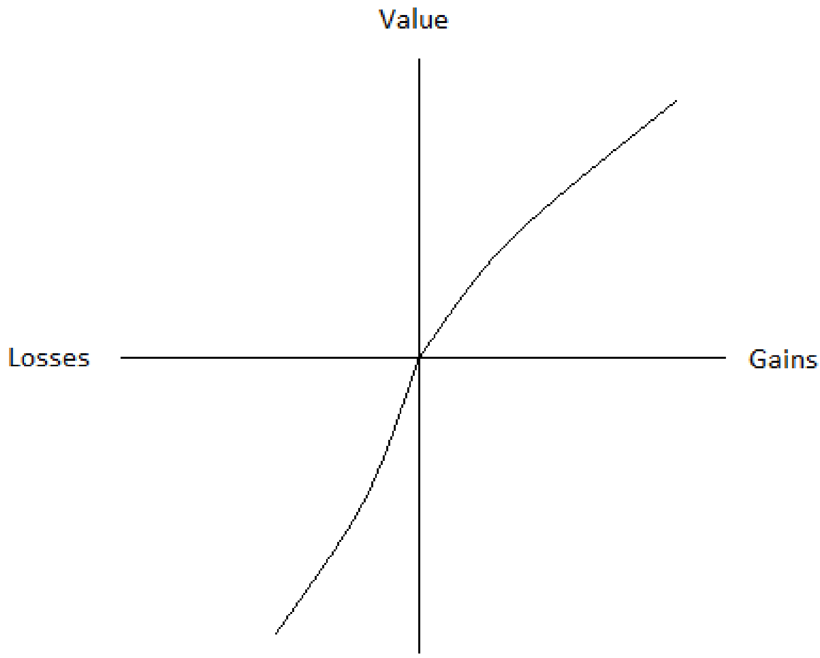
This type of model is called *rank-dependent*, since the probability weighting of an outcome depends on how it ranks in relation to the other outcomes in the prospect. The highest outcome, x, receives weight $w^+(p)$. The lowest outcome, y, receives weight $1 - w^+(p)$, which is different from the weigh of its probability, $w^+(1 - p)$. Gonzalez and Wu (1999) give an example of a prospect with equal chance of receiving \$100 or \$50, where an individual with $w^+(0,5) = 0,3$ assigns the weight of 0,3 to outcome \$100 and the weight of 0.7 to outcome \$50, although both outcomes have the same probability of occurring.

CPT conjectures that the value function v is concave for gains and convex for losses, implying that the impact of a variation in outcomes decreases with the distance from the reference point. For example, a change in gains from 10 to 20 has more impact in utility than a change from 100 to 110. Moreover, v is assumed to be steeper in the domain of losses ($v'(x) < v'(-x)$), which means that losses have greater impact on utility than similar gains. This last hypothesis is called *loss aversion* and it will be discussed in subsection 2.3 below. Figure 1, displayed in Kahneman and Tversky (1979), illustrates the usual format estimated for the value function.

The probability weighting function (pwf) has the unit interval as domain and image, but it is not necessarily linear in the probabilities. The sum of weights given to complementary events can be, for example, less than 1 (subadditivity). Another feature commonly observed is overweight ($w(p) > p$) of small probabilities and underweight ($w(p) < p$) of medium and large probabilities. Also, the impact of a change in probability decreases when the probability is far away from the two extremes, certainty ($p = 1$) and impossibility ($p = 0$). For example, an increase in the probability from 0% to 10% or from 90% to 100% is more significant than an increase from 50% to 60%. Thus, the probability weighting function is less steep around 0.5, concave near 0 and convex near 1. Examples of probability weighting functions w^+ and w^-, estimated in experiments, are illustrated in Figure 2.

The S format of the value function and the inverted-S format of the probability weighting function imply certain features in behavior, such as the fourfold pattern of risk attitudes (table 1 above). In expected utility theory, concavity and convexity of the utility function imply risk aversion and risk propensity, respectively. The same no longer holds in general for prospect theory. Suppose, as in the

Figure 1: Value Function



Source: Kahneman and Tversky (1979)

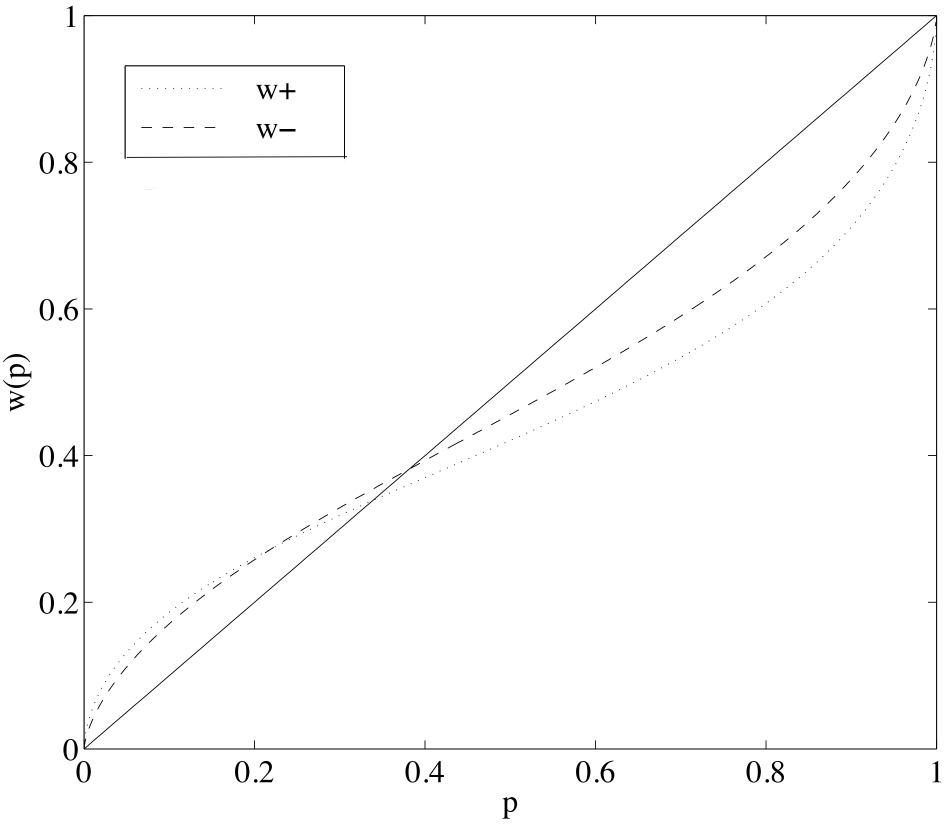
example given by Abdellaoui et al. (2007), someone indifferent between the lottery $(-\$100, 1/2\,; \$0)$ and a certain loss of \$ 40. Since the certainty equivalent is greater than the expected value of the lottery $(-40 > -50)$, then this choice displays a risk loving behavior. Still, it is possible that the utility function over outcomes is concave, provided that $w^{-}(1/2) < 0.4$.

The estimation of utility under prospect theory is complex. It is necessary to determine not only the transformation of outcomes, but also the transformation of probabilities into decision weights, besides how to combine these two functions. Some authors, like Gonzalez and Wu (1999) and Abdellaoui (2000), suggest non-parametric estimation methods, arguing that they do not depend on the assumption of functional forms. Others, such as Tversky and Kahneman (1992), Hey and Orme (1994) and Donkers et al. (2001) use parametric methods, which allow for an estimation that uses a small number of questions about preferences and responses carry no errors during the procedure.

Abdellaoui et al. (2008) propose an estimation method that combines the advantages of both parametric and non-parametric methods. They designed an experiment with a small number of fixed probabilities, transforming it in a parameter to be estimated. Thus, it is not necessary to make assumptions about the probability weighting function form and its corresponding coefficients. As for the value function over outcomes, a traditional functional form is used, whose fitness to data can be tested independently.

This method uses a relatively simple experiment, with only six lotteries to estimate the utility of prospects that involve gains and six for estimating the utility of prospects that involve losses. For each prospect we find its certainty equivalent (CE), that is, the value which renders the participant indifferent between a certain amount of money and the lottery. Bostic et al. (1990) show that directly asking in experiments the certainty equivalent of a lottery often leads to reversal of preferences: lotteries with lower certainty equivalents are preferred to lotteries with higher certainty equivalents. They argue that to elicit the CE through a series of choices generates fewer inconsistencies. Since the estimation

Figure 2: Probability Weighting Functions



Source: Kahneman and Tversky (1992)

method requires a small number of CE, we can use a relatively small number of choices without imposing a large cognitive burden on participants.

2.3 Loss Aversion

Observing people's decisions facing the possibility of having their wealth increased or decreased, Kahneman and Tversky (1979) noted that losses tend to have a greater impact on utility than gains. The pleasure in winning an amount of money is less than the discontent of losing the same amount. This result is called *loss aversion* and explains phenomena such as the high risk premium for stocks (Benartzi and Thaler, 1995) and the endowment effect – the difference between the willingness to pay and the willingness to sell a certain good (Thaler, 1980). To quantify loss aversion, we need to define it formally. However, there is no single way to do it, since loss aversion is a behavioral concept. This section discusses and compares some existing measures.

Kahneman and Tversky (1979) start from the idea that symmetrical lotteries, $(x, 1/2\,; -x)$, with $x > 0$, are not appealing to most people. Thus:

$$v(0) = 0 > w^{+}(1/2)v(x) + w^{-}(1/2)v(-x)\,.$$

Prospect theory postulates the reflection effect, which implies $w^{+}(1/2) = w^{-}(1/2)$, and then $v(x) < -v(-x)$. This suggests that the average $-v(-x)/v(x)$ can be used to calculate loss aversion at a given interval (Abdellaoui et al., 2007). Since probability weighting functions can be different for gains and losses, loss aversion may also be dictated by the format of w and not only by the format of v, as argued by Schmidt and Zank (2005). To account for this possibility, the previous formula can be modified to $w^{-}(1/2)v(-x)/w^{+}(1/2)v(x)$.

Tversky and Kahneman (1992) assume $v(x) = x^{\alpha}$ for $x \geq 0$ and $v(x) = \lambda[-(-x)^{\beta}]$ for $x < 0$, where λ is defined as the coefficient of loss aversion. Thus, $\lambda = -v(-1)/v(1)$. This measure captures the meaning of loss aversion when the parameters α and β of the value function for gains and losses are equal, as assumed by Kahneman and Tversky (1979). This *reflection effect* is usually supported by estimations with aggregated data. Individually, however, these parameters can be quite distinct, as found by Gonzalez and Wu (1999), Abdellaoui et al. (2007) and Laury and Holt (2008). In this case, the measure $\lambda = -v(-1)/v(1)$ does not capture well the meaning of loss aversion.

Kahneman and Tversky (1979) also argued that the aversion to symmetric lotteries generally increases with the amount at stake: for $x > y \geq 0$, $(y, 1/2\,; -y)$ is preferred to $(x, 1/2\,; -x)$. Thus:

$$w^{+}(1/2)v(y) + w^{-}(1/2)v(-y) > w^{+}(1/2)v(x) + w^{-}(1/2)v(-x)\,.$$

When $w^{+}(1/2) = w^{-}(1/2)$ and y approaches x, we have that $v'(x) < v'(-x)$, i.e., the value function is steeper in the loss domain. The average $v'(-x)/v'(x)$ indicates how much steeper it is and is another measure of loss aversion often used (Wakker and Tversky, 1993). Again, if we assume that the probability weighting function for gains and for losses are different, an adaptation of this measure proposed by Schmidt and Zank (2005) is $w^{-}(1/2)v'(-x)/w^{+}(1/2)v'(x)$. This measure defines loss aversion as the ratio of the impact of a variation in loss with a certain probability given the impact of that change in a gain of the same size and the same probability. This impact is given by the combined effect of the probability weighting function and the slope of the utility function.

Finally, Tversky and Kahneman (1991) estimate the coefficient of loss aversion by the ratio G/L, such that a lottery with equal chances of winning G or losing L becomes acceptable. The authors found in several experiments that this ratio is approximately 2:1. For example, a proposal to win \$25 with 50% of chance and losing \$10 with 50% of chance is usually the limit between being accepted and being refused. This measure typically vary with the size of the outcomes: increased loss is only accepted with a proportionately higher increase in gain.

The first three definitions lead to the same result under the assumptions of Kahneman and Tversky (1979), i.e., when the parameters of the value function for gains and losses are equal ($\alpha = \beta$) and when the reflection effect holds (implying that $w^+(1/2) = w^-(1/2)$):

$$\frac{-w(1/2)(-\lambda)x^\alpha}{w(1/2)x^\alpha} = \frac{-(-\lambda)1^\alpha}{1^\alpha} = \frac{-w(1/2)(-\lambda)\alpha x^{\alpha-1}}{w(1/2)\alpha x^{\alpha-1}} = \lambda\,.$$

Therefore, many authors compute loss aversion sometimes with one or another of these concepts, not making clear the different hypothesis beneath each measure. The last definition is equivalent to the previous ones with one more additional condition, that the value function is linear on outcome x:

$$w(1/2)G^\alpha - w(1/2)\lambda L^\alpha = 0 \quad \Rightarrow \quad \lambda = \frac{G^\alpha}{L^\alpha} \quad \Rightarrow \quad \lambda = \frac{G}{L}, \quad \text{if } \alpha = 1\,.$$

Table 2 illustrates how these definitions can lead to very different results when one of the above assumptions is not satisfied. The comparisons are made using the following hypothetical values for the parameters of the utility function[3]: $v(x) = x^{0.6}$ for $x \geq 0$, $v(x) = 0.5[-(-x)^{0.9}]$ for $x < 0$, $w^+(1/2) = 0.4$ and $w^-(1/2) = 0.45$. For each formula, if the coefficient is greater than 1, then there is loss aversion, otherwise there is loss propensity. The results obtained generally increase with x, i.e., the higher the value at risk, the greater the loss aversion. Therefore, to compare results of different studies it is necessary to standardize the range of x. In the example, results were computed for $x = 10$ and $x = 1,000$.

Table 2: Loss Aversion Values Computed Using Different Measures

Author and Definition	Formula	Example	
		$x = 10$	$x = 1,000$
Kahneman and Tversky (1979) modified: How bigger is the impact of losses relative to gains	$\frac{-w^-(1/2)v(-x)}{w^+(1/2)v(x)}$	1.122	4.468
Tversky and Kahneman (1992): How bigger is the impact of a \$1 loss relatively to a \$1 gain	$\frac{-v(-1)}{v(1)}$	0.500	0.500
Schmidt and Zank (2005): How much steeper is the value function for losses	$\frac{-w^-(1/2)v'(-x)}{w^+(1/2)v'(x)}$	1.684	6.702
Tversky and Kahneman (1991): G/L such that $(G, 1/2; L) \sim 0$	$\frac{G}{L}$	1.137	5.277

In summary, the coefficients obtained by different settings cannot be directly compared. It is important to standardize a measure of loss aversion that takes into account the valuation of outcomes

[3]The values assumed only illustrate the argument, but are consistent with values estimated in several articles.

and the weighting of different probabilities, for gains and losses, and to check if the assumptions that underpin each of these measures are valid.

As we discussed at the end of section 2.2, to estimate the utility of choices in risky situations, some studies specify functional forms and others use non-parametric methods. Each approach has advantages and weaknesses. Non-parametric estimations are independent of the choice of functional forms, meaning they are more general. However, estimates from these procedures might carry a large noise, which leads to a less accurate inference (Gonzalez and Wu, 1999). Parametric approaches usually have lower response errors and require a smaller number of questions. Its disadvantage is that a poor specification of the value function can bias the estimation of the probability weighting function and vice versa (Booij et al., 2010). We review now a few papers that estimate utility functions, focusing on the results they obtain. Table 3 below summarizes these results.

Tversky and Kahneman (1992) use $v(x) = x^{\alpha}$ as value function for gains ($x \geq 0$) and $v(x) = \lambda[-(-x)^{\beta}]$ as value function for losses ($x < 0$), where λ denotes the loss aversion coefficient. The probability weighting functional form used in their estimations is $w(p) = p^{\gamma}/[p^{\gamma} + (1-p)^{\gamma}]^{1/\gamma}$. They find a concave value function for gains and a convex value function for losses, with a coefficient equal to 0.88 in both domains. The values estimated for γ are 0.61 and 0.69 in the domains of gains and losses, respectively. The loss aversion coefficient estimated is 2.25.

Gonzalez and Wu (1999) non-parametrically estimate the value and probability weighting functions in an experiment for prospects with gains. They find that the value function is concave and the weighting function has an inverted S-shape, as predicted by prospect theory. However, their results show a great heterogeneity in individual parameters of the probability weighting function. Gonzalez and Wu (1999) argue that this function is better estimated using two parameters, $w(p) = \delta p^{\gamma}/[\delta p^{\gamma} + (1-p)^{\gamma}]$. The parameter γ reflects the pwf's curvature, and is related to the sensitivity of changes in the probability decreasing with greater distance from the extremes ($p = 0$ and $p = 1$). That is, γ indicates if the function approaches a linear shape or a reversed S-shape. The parameter δ reflects the pwf's height, and is related to the attraction of lotteries: if for one person $w(1/2) = 0.6$ and another $w(1/2) = 0.4$, a game with 50% gain is more attractive to the first person. The authors showed that the non-parametric estimates obtained fit well a the power functional form for the value function, $v(x) = x^{\alpha}$. Using nonlinear least squares, they estimate $\alpha = 0.49$.

Abdellaoui (2000) also suggests a non-parametric method to estimate the utility of prospects, obtaining a value function that is concave for gains and convex for losses. Their results also show that the probability weighting function for losses is more elevated than for gains, and in both domains there is an overweight of small probabilities. Abdellaoui et al. (2007) measure loss aversion at the individual level. They estimate the utility of gains and losses simultaneously in an experiment with choices between prospects. They find no significant difference between the probability weighting functions for gains and for losses. They also estimate a value function that is concave for gains and convex for losses. They show that the power value function fits well to their data, and estimate coefficients of 0.75 and 0.74 for gains and losses, respectively. Finally, loss aversion is measured as nearly 2, using the definition of Tversky and Kahneman (1991).

Booij and Kuilen (2009) non-parametrically estimate the utilities of a large sample of participants in an experiment. The coefficient of loss aversion is measured by the ratio between the gradient of the value function for gains over the gradient of the value function for losses. Their results also indicate a concave value function for gains and convex for losses. Under a parametric estimation with the same data, Booij et al. (2010) computed a coefficient of loss aversion of 1.6, using Tversky and Kahneman (1992) method. Furthermore, their data confirmed the S-inverted shape for the probability weighting function.

Table 3: Some Results

Paper	Values estimated						
	α	δ^+	γ^+	β	δ^-	γ^-	λ
$w(p)$ with one parameter:							
Tversky and Kahneman (1992)	0.88		0.61	0.88		0.69	2.25
$w(p)$ with two parameters:							
Gonzalez and Wu (1999)	0.49	0.77	0.44				
Abdellaoui (2000)	0.89	0.65	0.60	0.92	0.84	0.65	
Kilka and Weber (2001)		1.04					
Fehr-Duda et al. (2006)	1.14	0.82	0.52	1.05	1.04	0.53	
Booij et al. (2010)	0.86	0.77	0.62	0.83	1.02	0.59	1.58
Resende and Wu (2010)		1.05			1.02		
$w(p)$ non-parametric:							
Abdellaoui et al. (2007)	0.75			0.74			2.04^a
Booij and van de Kuilen (2009)							1.84^b

Values for the median. Value function: $v(x) = x^\alpha$ for $x \geq 0$ and $v(x) = -\lambda(-x)^\beta$ for $x < 0$. One parameter pwf: $w(p) = p^\gamma/[p^\gamma + (1-p)^\gamma]^{1/\gamma}$; two-parameter pwf: $w(p) = \delta p^\gamma/[\delta p^\gamma + (1-p)^\gamma]$. a: $\lambda = v(-x)/v(x)$; b: $\lambda = v'(-x)/v'(x)$.

Kilka and Weber (2001) estimate probability weighting functions with two parameters in a context of uncertainty, i.e., where prospects are associated with chance events of unknown probability. In this case, the weighting function depends on the probabilities estimated by the decision maker and the domain of uncertainty. The pwf's elevation parameter δ estimated is between 0.89 and 1.32 for events whose knowledge is high (small uncertainty). For events where there is little knowledge about their probability of occurrence, the same parameter is estimated between 0.79 and 1.17. Table 3 presents the average values for this parameter, equal to 1.04. The results obtained by Kilka and Weber (2001) show that lotteries for which participants have greater knowledge are more attractive.

Resende and Wu (2010) investigate if the results of Kilka and Weber (2001) can be extended to losses, testing if the *reflection effect* of Kahneman and Tversky (1979) holds in this situation. For gains and high knowledge, they estimate an elevation parameter δ between 1.07 and 1.27 and for gains and low knowledge, between 0.77 and 1.10. For losses, δ is estimated between 0.99 and 1.04 and they find no significant difference between low and high knowledge domains. These results are shown in Table 3 for the mean values: 1.05 to 1.02 for losses and gains. These authors conclude that in a situation of uncertainty, the effect of knowledge on the probability weighting function does not reflect well from gains to losses.

All these studies (and many others) point that probabilities are not linearly weighted in the utility, the probability weighting function has a S-inverted format, the value function has an S format, and the loss aversion coefficient is always higher than 1 (more specifically, in the range from 1.5 to 3). On conflicting results, the reflection effect is sometimes corroborated and sometimes not. A probability weighting function with two parameters seems better fitted to be used specially in estimations with individual data.

3 The Model

We use Abdellaoui et al. (2008) method to estimate a value function for gains and for losses at individual level. We fix a probability p_g for a series of prospects that involve gains, $(x_i, p_g\,; y_i)$, $x_i > y_i \geq 0$, $i = 1, \ldots, k$. The certainty equivalent G_i of prospect i satisfies:

$$u(G_i) = \delta^+[u(x_i) - u(y_i)] + u(y_i)\,, \tag{1}$$

where $\delta^+ = w^+(p_g)$. If we keep p_g fixed, there is no need to estimate the whole probability weighting function, but just one value of it. For the value function, we suppose:

$$v(x) = \begin{cases} u(x) & \text{if } x \geq 0 \\ \lambda u(x) & \text{if } x < 0, \end{cases} \tag{2}$$

where $u(x) = x^\alpha$ and λ is the loss aversion coefficient, under the definition of Tversky and Kahneman (1992). Therefore:

$$G_i = [\delta^+(x_i^\alpha - y_i^\alpha) + y_i^\alpha]^{1/\alpha}\,. \tag{3}$$

A similar procedure is done for prospects that involve losses: we fix a probability p_l and use a series of prospects $(x_i, p_l\,; y_i)$, $x_i < y_i \leq 0$, $i = 1, \ldots, k$, with certainty equivalents denoted by L_i. In this case we have:

$$\lambda u(L_i) = \delta^-[\lambda u(x_i) - \lambda u(y_i)] + \lambda u(y_i)\,, \tag{4}$$

where $\delta^- = w^-(p_l)$. For $x < 0$, we assume that $u(x) = -(-x)^\beta$. Thus:

$$-(-L_i)^\beta = \delta^-[-(-x_i)^\beta + (-y_i)^\beta] - (-y_i)^\beta\,,$$

which is equivalent to:

$$-L_i = \{\delta^-[(-x_i)^\beta - (-y_i)^\beta] + (-y_i)^\beta\}^{1/\beta}\,. \tag{5}$$

Once the certainty equivalents for each prospect are obtained in the experiment, we can estimate α and β without assuming anything about the probability weighting functional form. We simply estimate the value of δ.

We cannot identify the loss aversion parameter λ using (4), but once we compute α and β, it is possible to estimate λ using a mixed prospect. Lets denote by G^* a gain where u was determined in the first stage, and by L^* the loss that makes $(G^*, p_g\,; L^*) \sim 0$. If we let $p_l = 1 - p_g$, then we have:

$$\delta^+ u(G^*) + \delta^- \lambda u(L^*) = u(0) = 0\,. \tag{6}$$

The values of δ^+, $u(G^*)$, δ^- and $u(L^*)$ are obtained from our estimation of equations (3) and (5). Therefore, we can compute λ as:

$$\lambda = \frac{\delta^+ u(G^*)}{\delta^- u(L^*)}\,. \tag{7}$$

4 Experiment Design

The experiment was conducted with 24 undergraduate and 23 graduate students in Economics at the University of Brasilia in april 2012[4]. The goal is to compare the preferences of the two groups, testing if instructions and initial examples in the experiment are sufficient to generate consistent choices. Choices were computer-based, made in a research lab. Figure 3 illustrates a windows page of the program designed for the experiment. The software used to develop the program was SmallBasic 1.0 (12 June 2011, Copyright (c) Microsoft Corporation). Certainty equivalents were elicited through a series of binary choices, as described below. Within each question, a windows would ask the participant to confirm the choice made.

The experimental sessions lasted on average 36 minutes, with nine minutes used for instructions and examples. To encourage participants to choose the preferred alternative, they were told that at the end of the experiment, one prospect with gains, one prospect with losses and one mixed prospect would be randomly selected. In the first, they could earn up to R$ 10.00, and in the second they could miss up to the amount obtained in the first. In the mixed prospect, they could gain or lose more R$ 9.00. Participants never had to pay anything of their own money. One of the participants was sorted to compete for a prize on a larger scale, with the possibility of earning up to R$ 190.00. Non-monetary incentives were also used to attract participants. Undergraduates were asked to attend as part of the discipline Microeconomics 3, and their sessions were held on the same day for all participants of this group, during class time. Graduates also participated in a single day, and received 5 extra points in the grade of the first exam (with a total value of 100 points) of the discipline Microeconomics 1.

To estimate the value function, we used two different probability values, $p_g = 1/2$ and $p_g = 2/3$, similarly to Abdellaoui et al. (2008). Under prospect theory, the value function is independent of probabilities, which implies that estimation of $u(x)$ with each value of p should not generate significant differences. Since $p_l = 1 - p_g$ for losses lotteries, we use $p_l = 1/2$ and $p_l = 1/3$, since we need these values matched to be able to compute λ from the mixed prospects.

[4]As an illustration, Abdellaoui et al. (2008) gathered data from 47 students graduate in Economics and Mathematics from the Ecole Normale Supérieure, Antenne de Bretagne, France.

Figure 3: A Windows from the Experiment

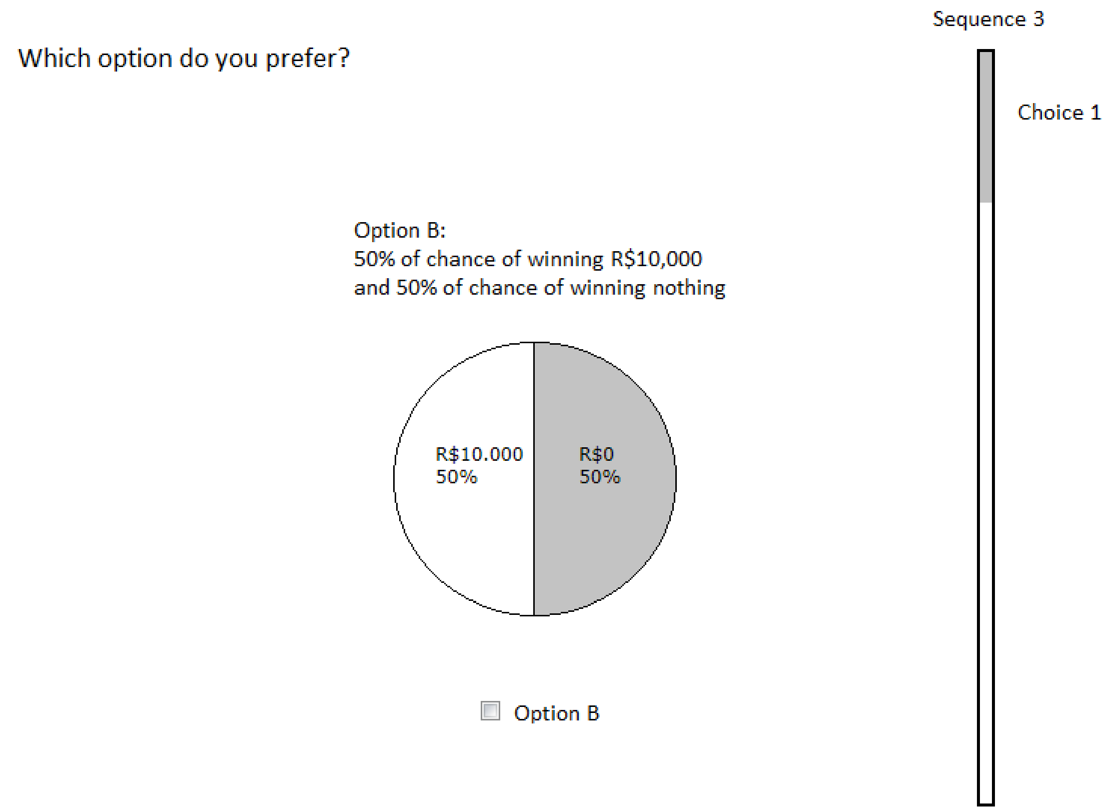


Table 4 presents the values offered in the series of prospects for which the certainty equivalents were elicited. For each fixed probability value, we used six prospects with gains, $(x_i, p_g\,; y_i)$, $i = 1, \ldots, 6$, and six prospects with losses, $(x_i, p_l\,; y_i)$, $i = 1, \ldots, 6$. Outcomes of prospects with losses are equal in absolute value to outcomes of prospects with gains. The values chosen were high, in order to test if results would differ from other studies, which commonly use small values. Unfortunately it was not possible to use real incentives equally high, and values received by participants were equal to the prospect's face value divided by one thousand. For a randomly selected participant, the value received was the prospect's face value divided by one hundred.

Similarly to Abdellaoui et al. (2008), prospects were randomly presented to participants[5], with gains prospects presented first and then losses prospects. Abdellaoui et al. (2008) observed, in trial experiments, that people found it easier to start with gains. Therefore, the program always started with them. Following the elicitation of the 24 certainty equivalents, six questions, randomly chosen for each participant, were repeated, in order to observe the rate of error in responses. Then, a series of six mixed prospects $(g_i^*, 1/2\,; L_i^*)$, $i = 1, \ldots 6$, were presented to estimate the loss aversion coefficient λ. Although only one mixed prospect is required to estimate λ, six were used in order to test whether the estimation of λ varied between them. The value chosen for G_i^* was the expected value of the lotteries $(x_i, 1/2\,; y_i)$, $i = 1, \ldots, 6$: 1,000, 2,000, 3,000, 5,000, 8,000 and 9,000, respectively. The goal was to determine L_i^* such that $(g_i^*, 1/2\,; L_i^*) \sim 0$.

Table 4: Lotteries Values used in the Experiment

	Index i					
	1	2	3	4	5	6
$\lvert x_i \rvert$	2,000	4,000	6,000	10,000	10,000	10,000
$\lvert y_i \rvert$	0	0	0	0	6,000	8,000

Values in Reais (R$).

The method of binary choices is illustrated in Table 5. The first example shows how to find the certainty equivalent of prospect $G_2 = (4,000, 1/2\,; 0)$. The first choice is between G_2 and its expected value, R$ 2,000. The alternatives chosen in the example are in bold. If the participant prefers the prospect, the value offered with certainty increases in the next choice window. Otherwise, the value decreases. The size of the variation in value is given by $(x_i - y_i)/2^n$, where n is the iteration's number. That is, the variation is always half the former, in order to make the value offered converge to the certainty equivalent. In the example, the value varies by R$ 1,000, R$ 500, R$ 250 and R$ 125, sequentially. Thus, we obtain a range in which the certainty equivalent belongs and assume that it is equal to the average of this range. For example, if choices led to a value between R$ 2,625 and R$ 2,750, the certainty equivalent is assumed to be R$ 2,687.50.

The third column of Table 5 shows an example of the procedure for mixed lotteries, used to compute the coefficient of loss aversion. The goal here is to determine the loss L_6^* that makes the prospect $(9,000, 1/2\,; L_6^*)$ indifferent to R$ 0. Six choices are presented. If the participant prefers the lottery, the negative value decreases in the next question. If he prefers not to participate in the lottery, the value increases, approaching zero. In the end, the value of the loss L_6^* is computed as the average of the range the choices made led to. In the example in table 5, the range obtained is from -1687 to -1405, and thus it is assumed that L_6^* is equal to R$ -1,546.

[5] The choice questions were scrambled using the Fisher-Yates shuffle algorithm (Fisher and Yates, 1938; Durstenfeld, 1964).

Table 5: An Illustration of the Binary Choice Method

Interaction	Elicitation of G_2	Elicitation of L_6^*
1	2,000 vs **(4,000, 1/2; 0)**	**0** vs. (9,000, 1/2; -9,000)
2	**3,000** vs (4,000, 1/2; 0)	**0** vs. (9,000, 1/2; -4,500)
3	2,500 vs **(4,000, 1/2; 0)**	**0** vs. (9,000, 1/2; -2,250)
4	**2,750** vs (4,000, 1/2; 0)	0 vs. **(9,000, 1/2; -1,125)**
5	2,625 vs **(4,000, 1/2; 0)**	**0** vs. (9,000, 1/2; -1,687)
6		0 vs. **(9,000, 1/2; -1,405)**
Value Computed	G_2 = 2,687.50	L_6^* = -1,546.00

5 Results

Choices showed a good reliability rate: 78% of them remained unaltered in the six questions repeated at the end of both series of gains and losses. Thus, the preferences' rate of reversal, i.e., the percentage of different responses given by a participant to the same question was 22%, on average. Stott (2006) observes that rates of reversion are usually between 10% and 30% in the literature. However, different studies involve different experiments and thus these rates are not always directly comparable. Our results gave further support to the reliability of choices made by undergraduate students, when compared to graduate students: 80% and 76% of responses remained the same in each group, respectively.

The reliability rates presented above supports the validity of our results for undergraduates. Furthermore, the t-statistics of the coefficients estimated provide evidence that the experiment had understandable instructions and initial examples. The median t-statistic is 3.07 for gains prospects and 3.73 for losses prospects, for undergraduates. For graduate students, the values are 4.48 and 3.39, respectively.

5.1 Certainty Equivalents and Behavior under Risk

Results are presented for the 47 participants following Abdellaoui et al. (2008), for the ease of comparison. Table 6 shows the median and interquartile range of certainty equivalents found for each prospect.

For gains, the certainty equivalents' median for all participants is less than the expected value for the twelve lotteries, indicating risk aversion in the aggregate. Regarding individual results, 75% of certainty equivalents are smaller than the expected value of prospects with $p_g = 1/2$ and 74% for prospects with $p_g = 2/3$. For losses, seven prospects have median certainty equivalent lower than expected value, and higher for the other five. Among choices with $p_l = 1/2$, 59% show risk propensity and among those with $p_l = 1/3$, 50%. Thus, behavior in the domain of gains is fairly consistent with risk aversion and, in the domain of losses, although less homogeneous, it is consistent with risk loving. This result is similar to Abdellaoui et al. (2008) findings.

Table 7 classifies participants according to their attitude to risk. Part "a" of this table considers the same response error rate as in Abdellaoui et al. (2008). If in at least 8 of the 12 series of gains (losses), the certainty equivalent was less than the prospect's expected value, the participant was labeled risk averse. If the certainty equivalent was greater than the prospect's expected value in at least eight series, the participant was labeled risk lover. In all other cases, they were labeled as having a mixed behavior. Rates of reversal around 1/3 are also used in Abdellaoui et al. (2007), Fennema and

Table 6: Certainty Equivalents

Series	Gains: Expected Value	$p_g = 1/2$	Expected Value	$p_g = 2/3$
1	1,000	782 (594, 969)	1,333	1,105 (898; 1,313)
2	2,000	1,688 (938; 2,063)	2,667	2,375 (1,958; 2,709)
3	3,000	2,532 (1,969; 2,907)	4,000	3,313 (2,438; 4,063)
4	5,000	3,907 (2,344; 4,844)	6,667	4,896 (4,063; 6,563)
5	8,000	7,938 (7,438; 8,063)	8,667	8,375 (7,708; 8,709)
6	9,000	8,907 (8,719; 9,032)	9,333	8,980 (8,856; 9,272)
	Losses			
		$p_l = 1/2$		$p_l = 1/3$
1	-1,000	-906 (-1,094; -719)	-667	-603 (-769; -353)
2	-2,000	-2,063 (-2,438; -1,438)	-1,333	-1,374 (-1,706; -874)
3	-3,000	-3,094 (-3,656; -2,156)	-2,000	-2,063 (-2,813; -1,438)
4	-5,000	-5,156 (-6,094; -2,969)	-3,333	-3,436 (-4,061; -1,561)
5	-8,000	-7,563 (-8,063; -6,938)	-7,333	-7,040 (-7,540; -6,707)
6	-9,000	-8,906 (-8,969; -8,469)	-8,667	-8,686 (-8,769; -8,603)

Median values, with interquartile percentile between parentheses.

Assen (1999) and Etchart-Vincent (2004). The literature bases this rate on similar experiments about choices. Still, it is an arbitrary value. Therefore, Table 7, part "b" classifies participants considering at least 9 of the 12 series. That is, the second part considers an error margin of 1/4, closer to the 22% reversal rate encountered in our study.

Table 7: Individual Classification in relation to Risk Attitude

a) Reversal ratio of 1/3:

		Losses			
		Risk Aversion	Risk Propensity	Mixed	Total
Gains	Risk Aversion	18	13	7	38
	Risk Propensity	2	5	0	7
	Mixed	0	2	0	2
	Total	20	20	7	47

b) Reversal ratio of 1/4:

		Losses			
		Risk Aversion	Risk Propensity	Mixed	Total
Gains	Risk Aversion	12	11	13	36
	Risk Propensity	1	4	1	6
	Mixed	1	4	0	5
	Total	14	19	14	47

In the domain of gains, participants are predominantly risk averse. In the domain of losses, findings are mixed, considering a reversion rate of 1/3, and it shows a high propensity for risk considering a reversion rate of 1/4. The significant number of participants with mixed behavior is linked to a new behavioral feature observed in the experiment: the attitude toward risk depends not only on the prospect value, but also on its format. For $p_g = 1/2$, 18 participants displayed a risk loving attitude in the series where a gain was guaranteed (numbers 5 and 6). When there was no gain guaranteed (series 1-4), choices displayed risk aversion. For example, the certain gain of R\$ 2,000.00 is usually preferred to lottery $(4,000, 1/2\,; 0)$, indicating risk aversion. For the same participant, the lottery $(10,000, 1/2\,; 6000)$ is often preferred to the certain gain of R\$ 8,000, a risk loving behavior. When there is the possibility in the lottery of no gain whatsoever, the participant tend to be risk averse. But when there is a gain guaranteed in the lottery, some individuals are willing to take more risk, in order to get the additional gain. A similar phenomenon happens for lotteries with losses. In the series of losses, 14 participants altered the attitude toward risk in the two series in which there is a certain minimum loss (series 5 and 6). What seems to make these participants display a risk loving behavior in the first four series is the possibility of not losing anything. When a minimum value becomes a certain loss, as it happens in the last two series, some participants change to a risk averse behavior.

As we discussed in section 2.2 above, Kahneman and Tversky (1979) argued that choices depend on the presentation of the prospect. Here, we find a new type of frame effect. The prospect that proposes winning R\$ 8,000 to participate in the lottery $(10,000, 1/2\,; 6,000)$ is equivalent in terms of final outcomes to a prospect that pays first R$ 6,000 and proposes winning R$ 2,000 or participate in the lottery $(4,000, 1/2\,; 0)$. Kahneman and Tversky (1979) argue that due to a *cancelation effect*, this last game is usually analyzed disregarding the common component of R$ 6,000. Preferences over this lottery should be the same as the one that proposes winning R$ 2,000 or participate in the lottery $(4,000, 1/2\,; 0)$. However, this is not the result found by observing the choices made in series 2 and 5. Thus, the attitude toward risk clearly depends on how lotteries are formed. A certain portion of income guaranteed in the lottery can reverse the attitude toward risk, inducing a frame effect.

If we assume a power value function, what usually is done in most parametric estimations of utility, this behavior restricts the probability weight value $\delta = w^+(1/2)$. The first choice implies that $8^\alpha < 6^\alpha + \delta^+(10^\alpha - 6^\alpha)$, while the second choice implies that $2^\alpha > \delta^+ 4^\alpha$. The two inequalities cannot be simultaneously satisfied for a value of $\delta = w^+(1/2)$ lower or equal to 0.5. That is, choices cannot be explained by expected utility theory ($\delta^+ = 0.5$) and cannot be explained by prospect theory if the 1/2-probability is supposed to be underweighted ($\delta^+ < 0.5$). For individuals that make these choices, the 1/2-probability must be overweighted.

5.2 Parameters Estimated

The parameters were estimated by nonlinear least squares using data from each participant and also using aggregate data. First, we present results for each individual. Table 8 shows the median of individual coefficients for the value function and the weighting probability function. As a first test of the validity of prospect theory, the coefficients of the utility function obtained with different probabilities are compared using Wilcoxon test. For prospects that involve gains we found no significant difference between the estimated values of α under both probability values: $p_g = 1/2$ and $p_g = 2/3$ (p-value $=$ 0.735), consistent with prospect theory. For losses, the difference in the estimates of β with $p_l = 1/2$ and $p_l = 1/3$ is more pronounced (p-value $= 0.182$), but still not significant at 10%.

The remainder of this section focuses on the results obtained with $p_g = p_l = 1/2$, as in Abdellaoui et al. (2008). These authors reported a value of 0.86 for α and of 1.06 for β. The values we estimated for these parameters are 0.89 and 1.24, respectively. The difference between the losses parameters between the two studies might be related to the scheme of incentives we use: while Abdellaoui et al. (2008) provided incentives only for prospects with gains, our experiment also provided incentives for choices involving losses. It is possible that this has led to the increased risk aversion observed here for losses. Holt and Laury (2002) also found increased risk aversion using real incentives, corroborating this view.

Regarding the values estimated for the probability weights, we have $\delta^+ = w^+(1/2) = 0.48$ and $\delta^- = w^-(1/2) = 0.41$, similar to the values estimated by Abdellaoui et al. (2008): $\delta^+ = w^+(1/2) = 0.46$ and $\delta^- = w^-(1/2) = 0.45$. The weights underestimate the 1/2-probability. We reject the hypothesis that $\delta^+ = w^+(1/2) = 1/2$, with p-value equal to 0.004. For the hypothesis $\delta^+ = w^+(2/3) = 2/3$, we find that this probability is underweighted, with p-value $= 0.000$. For losses, the 1/2 the 1/3 probabilities are both underweighted, with p-values of 0.000 and of 0.024, respectively. Finally, the hypothesis that the weighted probability 1/2 is the same for gains and losses domains, i.e. that $w^+(1/2) = w^-(1/2)$, cannot be rejected, with p-value $= 0.186$. Thus, there was no clear objection to the use of the same weighting for the 1/2-probability for gains and for losses.

Table 8: Estimated Median of the Parameters

	α	δ^+		β	δ^-
$p_g = 1/2$	0,89	0,48	$p_l = 1/2$	1,24	0,41
$p_g = 2/3$	0,99	0,55	$p_l = 1/3$	1,01	0,28

Table 9: Individual parameters estimated using MQNL (undergraduate students).

	(Gains)		(Losses)	
Participant Number	α	δ^+	β	δ^-
1	1.00 (0.00)	0.48 (0.00)	0.69 (0.31)	0.14 (0.11)
2	1.00 (0.07)	0.40 (0.02)	0.99 (0.26)	0.53 (0.08)
3	0.70 (0.20)	0.25 (0.09)	1.70 (0.80)	0.44 (0.16)
4	1.54 (0.77)	0.55 (0.15)	0.80 (0.23)	0.22 (0.09)
5	0.71 (0.08)	0.52 (0.04)	0.92 (0.20)	0.52 (0.07)
6	0.79 (0.26)	0.50 (0.11)	2.09 (0.58)	0.45 (0.09)
7	1.14 (0.17)	0.40 (0.05)	1.69 (0.27)	0.22 (0.05)
8	0.98 (0.39)	0.43 (0.13)	1.00 (0.26)	0.57 (0.08)
9	1.03 (0.39)	0.43 (0.13)	1.57 (0.95)	0.48 (0.20)
10	1.66 (0.89)	0.49 (0.17)	3.10 (0.76)	0.17 (0.07)
11	0.56 (0.23)	0.51 (0.13)	1.07 (0.08)	0.48 (0.03)
12	2.72 (1.14)	0.50 (0.13)	0.83 (0.26)	0.21 (0.09)
13	0.64 (0.48)	0.37 (0.26)	3.44 (1.60)	0.20 (0.14)
14	0.71 (0.05)	0.51 (0.02)	2.39 (1.01)	0.36 (0.14)
15	0.86 (0.09)	0.37 (0.04)	0.75 (0.15)	0.43 (0.07)
16	0.61 (0.28)	0.49 (0.15)	1.03 (0.88)	0.62 (0.24)
17	0.58 (0.19)	0.53 (0.10)	1.61 (0.31)	0.44 (0.06)
18	0.66 (0.22)	0.57 (0.10)	0.65 (0.31)	0.67 (0.12)
19	0.81 (1.28)	0.87 (0.19)	0.47 (0.23)	0.41 (0.16)
20	1.97 (0.51)	0.24 (0.08)	1.98 (0.39)	0.24 (0.06)
21	0.54 (0.08)	0.40 (0.05)	1.35 (0.35)	0.48 (0.08)
22	0.93 (0.35)	0.46 (0.13)	1.47 (0.25)	0.39 (0.06)
23	2.09 (0.75)	0.30 (0.12)	1.74 (0.29)	0.37 (0.06)
24	0.40 (0.08)	0.62 (0.05)	2.24 (1.45)	0.40 (0.22)

Standard-deviation under parentheses.

Table 10: Individual parameters estimated using MQNL (graduate students).

	(Gains)		(Losses)	
Participant Number	α	δ^+	β	δ^-
25	1.42 (0.43)	0.24 (0.10)	1.34 (0.23)	0.46 (0.06)
26	1.00 (0.00)	0.48 (0.00)	1.00 (0.00)	0.52 (0.00)
27	2.72 (1.58)	0.05 (0.08)	1.61 (0.65)	0.29 (0.14)
28	0.90 (0.00)	0.55 (0.00)	1.26 (0.47)	0.50 (0.12)
29	0.35 (0.05)	0.49 (0.05)	1.38 (0.81)	0.21 (0.18)
30	0.76 (0.27)	0.31 (0.12)	0.63 (0.18)	0.25 (0.09)
31	0.61 (0.22)	0.45 (0.12)	4.10 (1.38)	0.12 (0.08)
32	0.78 (0.06)	0.49 (0.03)	0.99 (0.10)	0.52 (0.03)
33	1.06 (0.08)	0.51 (0.03)	0.63 (0.25)	0.43 (0.14)
34	0.41 (0.05)	0.52 (0.04)	2.93 (2.27)	0.18 (0.22)
35	1.00 (0.00)	0.52 (0.00)	1.00 (0.00)	0.48 (0.00)
36	2.22 (0.41)	0.16 (0.05)	1.66 (0.43)	0.41 (0.09)
37	0.79 (0.16)	0.04 (0.02)	0.52 (0.65)	0.95 (0.06)
38	0.70 (0.25)	0.55 (0.11)	2.20 (1.37)	0.39 (0.21)
39	3.52 (2.84)	0.18 (0.23)	1.32 (0.61)	0.65 (0.12)
40	1.09 (0.29)	0.18 (0.08)	0.71 (0.15)	0.32 (0.07)
41	0.74 (0.17)	0.53 (0.07)	1.22 (0.36)	0.43 (0.10)
42	0.85 (0.28)	0.54 (0.10)	0.93 (0.18)	0.54 (0.06)
43	1.98 (0.37)	0.36 (0.06)	1.17 (0.08)	0.40 (0.02)
44	1.21 (0.17)	0.40 (0.05)	1.28 (0.26)	0.32 (0.07)
45	1.20 (0.73)	0.17 (0.17)	0.88 (0.52)	0.25 (0.19)
46	0.72 (0.32)	0.43 (0.15)	0.92 (0.34)	0.09 (0.07)
47	1.43 (0.69)	0.52 (0.15)	1.25 (0.21)	0.29 (0.06)

Standard-deviation under parentheses.

Table 9 presents the individual parameters estimated for undergraduate students and Table 10 presents them for graduate students. These tables show the large variation between individuals, as Abdellaoui et al. (2008) found, indicating strong heterogeneity in people's choices under risk. Undergraduate students had a higher curvature of the value function for both gains and for losses. The median value of α and β were 0.83 and 1.41, for undergraduate students, and 0.99 and 1.22, for graduate students. Since the estimates of the probability weighting function were similar between the two groups, the higher curvature of the value function indicates increased risk aversion for undergraduate students, when compared to graduates.

Table 11 classifies participants according to the shape of their value function for gains and for losses. It is concave (convex) if α is lower (higher) than 1. For losses, it is convex (concave) if β is less (greater) than 1. Most participants have a concave value function for both gains and losses. Comparing these results with those in Table 7, we observe that the concavity of the utility function does not necessarily imply risk aversion. In the case of prospects involving losses, while 29 participants have concave utility function, only 20 participants were classified as risk averse. Chateauneuf and Cohen (1994) showed that under prospect theory an individual may be risk loving and yet present decreasing marginal utility of wealth. For losses, it is sufficient that the probability weight be small, as pointed out in section 2.2. This result corroborates that it is not necessary that the value function for losses be convex to obtain the risk loving behavior often observed in this domain.

Table 11: Utility Function Shape

		Losses		
		Concave	Convex	Total
	Concave	17	9	26
Gains	Convex	12	9	21
	Total	29	18	47

Table 12 presents the parameters estimated using aggregated data. Since there are six observations for each participant, we show in parentheses the cluster-robust standard errors. The estimated value function is concave for both gains and for losses. For $p = 1/2$, the estimated value of α is 0.94 and of β is 1.27. Abdellaoui et al. (2008) estimated 0.81 and 1.19 for gains and losses, using aggregated data.

We also check if parameters vary with education. In the domain of gains, the estimated parameters of the value function for undergraduates and graduate students are 0.89 and 1.00, respectively. The null hypothesis that coefficients are equal is rejected with p-value 0.034, using the maximum likelihood ratio test. This indicates that the preferences of the two groups of participants are significantly different and indicates the need to expand the target population of experiments on lotteries, utility and risk, in order to bring better information about the behavior under risk of the overall population. For the loss domain, we could not reject that the estimated parameters in each group are equal (p-value = 0.653).

Finally, when considering gender, the estimation for prospects with gains shows a slight increased risk aversion for women: α is 0.95 for men and 0.92 for women, being significantly different at 10% level (p-value = 0.056). In the losses domain, there was no significant difference in the curvature parameters of both gender groups (p-value = 0.809).

Table 12: Parameter estimated under MQNL with Aggregate Data

	α	δ^+		β	δ^-
$p_g = 1/2$	0.94 (0.14)	0.43 (0.04)	$p_l = 1/2$	1.27 (0.21)	0.39 (0.04)
$p_g = 2/3$	0.92 (0.21)	0.55 (0.05)	$p_l = 1/3$	1.13 (0.14)	0.29 (0.03)

Standard deviation in parentheses.

5.3 Loss Aversion

For mixed lotteries we find generally large risk aversion. Table 13 shows the median values for L_j^* such that $(G_j^*, 1/2\,; L_j^*) \sim 0$, $j = 1, \ldots, 6$. Regarding individual results, 31 participants were risk averse in all lotteries and 5 participants were risk averse in 5 of the 6 prospects. Only 5 participants were risk lovers in all series, and the other six participants made mixed choices.

If we substitute the parameters estimated for α and β in equation (6), we can compute the loss aversion coefficient λ. Doing this, we find that λ is between 0.77 and 1.16. Despite some variation between the values of λ obtained in each series, the null hypothesis that the six coefficients are equal could not be rejected by the Friedman test, with p-value of 0.887. Thus, the result is consistent with prospect theory.

Table 13: Mixed Lotteries

Srie	G^*	L^*	λ
1	1,000	-765 (-1,015, -390)	0.777
2	2,000	-1,281 (-1,969, -719)	0.905
3	3,000	-1,640 (-2,859, -890)	1.038
4	5,000	-2,265 (-3,984, -859)	1.053
5	8,000	-4,875 (-6,375, -1,625)	1.161
6	9,000	-4,359 (-6,890, -1,828)	1.028

Median Values and Interquantile interval in Parentheses.

In the first two series, λ lower than 1 indicates a propensity to loss, according to the definition of Tversky and Kahneman (1992), also used by Abdellaoui et al. (2008). In the last four series, λ indicates a small degree loss aversion. However, there is strong loss aversion in mixed lotteries. According to Tversky and Kahneman (1991), we can measure it as the ratio G_j^*/L_j^*. The median values for this ratio for each prospect are presented in the first column of Table 14. In all series, the loss that makes the individual indifferent between participating in the lottery and to remain in the same situation is much lower than the gain: between 1.31 and 2.21 times lower.

Table 14 also presents the median the three other measures of loss aversion discussed in section 2.3. The values of x used to compute these measures are the prospects' expected values. All these measures confirm the presence of loss aversion in every series. The highest values were found for loss aversion computed as the slope of the value function, in the third column, with the exception of the third series.

Table 14: Loss Aversion Related Measures

Series	$\dfrac{G^*}{L^*}$	$\dfrac{-\delta^- v(-x)}{\delta^+ v(x)}$	$\dfrac{-\delta^- v'(-x)}{\delta^+ v'(x)}$
1	1.31	1.49	1.68
2	1.56	1.67	1.80
3	1.83	2.15	2.05
4	2.21	2.31	3.29
5	1.64	2.32	2.42
6	2.06	2.53	2.62

Median values.

The estimates under each setting are distinct. As mentioned before, these estimates would only coincide if the curvature parameters were similar in gain and loss utility functions. In that case, each measure would present the same amount, although representing different traits. The measure in the first column of Table 14 reflects how much greater is the gain needed to compensate a certain loss. The advantage of this measure is that it does not require the estimation of the utility function, and it can be computed using only one question. The measure in the second column consists in the best approximation to the idea of loss aversion typically used: losses have a greater impact in relation to gains ("*losses loom larger than gains*", Kahneman and Tversky, 1979, p.279). The measure in the third column expresses the variation in perception of the loss relative to the gains, that is, how much greater is the impact of a marginal variation in a loss than a marginal variation in a gain. It is also an important measure to understand the decision-making process and it should be reported as an accessory parameter to measure loss aversion.

The values obtained by the three measures differ substantially from the values computed for λ in all series. Thus, λ is a parameter of the utility function that do not reflect loss aversion. This happens because the value and probability weighting functions are different for gains and losses, as discussed in section 2.3. This result is displayed in Table 8, wherein the coefficients of the value function are 0.89 and 1.24 for gains and losses, respectively. The weighting function was not significantly different between the domains of gains and losses. Still, it is important to take it in account when computing the individual coefficient of loss aversion, since for some participants this difference is large (Tables 9 and 10).

6 Conclusion

This study uses the method proposed by Abdellaoui et al. (2008) to measure the utility of prospects under gains and losses and the loss aversion of participants in a simple experiment. Our choice of the experimental procedure was motivated by its efficiency and its relative simplicity. Efficiency refers to significant results, consistent with prospect theory.

We departed from Abdellaoui et al. (2008) in three main aspects. First, we propose an incentive system that also encompasses prospects that involve losses. After all choices were made, a lottery with gains was drawn and the prize would be awarded to the participant. The, a lottery with losses would be drawn and the participant could lose part or the totality of what he or she had won. The goal was to induce a real sense of loss, not just a smaller gain, in a way to depart from the usual system of incentives and to avoid the so-called *house-money effect*.

The parameters estimated for prospects with gains were quite close to those found by Abdellaoui et al. (2008), while the parameters estimated for prospects with losses indicate an higher degree of risk aversion. It is possible that this difference is consequence of this scheme of incentives for losses, absent in Abdellaoui et al. (2008). Of course, it is also possible that other factors are responsible for this result, such as differences in culture and education of the participants. This point is worth of further research.

Second, the experiment was conducted not only with graduate students in economics, as in Abdellaoui et al. (2008), but also with undergraduates. The aim was to expand the sample of participants, which typically is limited to individuals with high education, due to the complexity of these type of experiments. This expansion brings new information about people's behavior under risk. We found a significant difference between the curvatures of the utility function of the two groups, highlighting the importance of broadening the scope of the trial population.

Third, we measured loss aversion in three different ways. The measure commonly used in parametric studies, as in Tversky and Kahneman (1992), corresponds to a parameter of the utility function for losses. When the curvature of the utility function is similar to gains and losses, this parameter reflects how much greater is the weight of losses relative to gains. However, the parameters of the curvature of the value function we found for lotteries of gains and losses were quite different: 0.89 and 1.24, respectively. In this case, other forms should be employed to measure loss aversion, in a way more compatible with its theoretical definition.

Our results also showed two features in the behavior of a significant number of participants that have important implications for prospect theory. The first confirms the prediction of Chateauneuf and Cohen (1994) and Schmidt and Zank (2005), that the propensity to risk when choices involve losses does not necessarily imply the convexity of the value function in the loss dimension. The choices displayed in our experiment showed that some participants made choices prone to risk, but had a concave value function. In such cases, risk propensity is due to the underweighting of the 1/2-probability, not because the parameter of the value function.

The other feature observed is that risk aversion and risk propensity are not unequivocally linked to gains and losses, respectively. Often an individual is risk averse in the gains domain when there is the possibility of winning nothing. The same individual can turn to be prone to risk when there is a certain minimum gain. The satisfaction of winning a certain value seems to induce some people to take risks when there is a chance of an additional gain. Similarly, some people are prone to the risk in the domain of losses when there is a chance of not losing anything. When there is a minimal loss for sure, they become risk averse over an additional loss. The dissatisfaction of not winning anything and the relief of not losing anything seem to have an effect in the decision-making process. When these possibilities are cut from the game, the attitude to risk can be reversed. With a power value function as normally assumed in the literature, this behavior can only be explained by prospect theory if the probability weighting function is overweighted in the probability used.

Finally, we find evidence about the heterogeneity of behavior between people, even more relevant because it relates to a very limited sample of the population: students of Economics, either undergraduates or graduates. Possibly, a similar study with a less homogeneous group of participants would lead to an even greater variation in the parameters estimated. Thus, this study emphasizes the importance of individual estimations and estimations with aggregate data, but with parameters dependent on other variables. These variables can be gender, income, education. As an example, we find an increased risk aversion for women and graduate students, when compared to men and undergraduate students, respectively.

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