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Hyperopic Strict Topologies

Jaime Orillo and Rudy José Rosas Bazán

Catholic University of Brasilia, Pontificia Universidad Católica del Perú and Instituto de Matemática y Ciencias Afines

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Jaime Orrillo†
Rudy José Rosas Bazán‡

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Abstract

We explicitly define a family of seminorms on the space of all bounded real sequence l^∞. This family gives rise to a Hausdorff locally convex topology which is not equivalent to the usual ones: the weak topology $\sigma(l^\infty, l^1)$, the norm topology τ_∞, the Mackey topology $m(l^\infty, l^1)$ and the strict topology β. We show that this new topology captures a hyperopic behavior of individuals[1]. We call this "hyperopic strict topology". Finally, we show that the hyperopic strict dual of l^∞ is not l^1 anymore but rather the set of all finitely additive measures. An interpretation of this hyperopic strict dual is offered in a convenient way.

Keywords: Hyperopic agents; locally convex topology; charges.
JEL D11, C02.

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†Catholic University of Brasilia. Graduate Program in Economics, SGAN 916, office A-116, Asa Norte, Brasilia - DF, Zip: 70.790-160. Brazil. Phone: +55(61)3448-7192. Fax: +55(61)3347-4797. Email: orrillo@pos.ucb.br.

‡Pontificia Universidad Católica del Perú and Instituto de Matemática y Ciencias Afines (IMCA). E-mail: rudy.rosas@pucp.pe

[1]That is, our interest is with agents who only see the distant future by completely neglecting any finite consumption stream.

1 Introduction

The linear space, l^∞, of all bounded real sequences has been used in economy, among many other things, to model the consumption space of individuals. They are assumed to rank all their consumption plans which are represented by elements of l^∞_+, this last being the consumption set.

Mathematically, on this space l^∞, many topologies have been defined. Among the most well-known we find: the weak topology $\sigma(l^\infty, l^1)$, the norm topology τ_∞, the Mackey toplogy $m(l^\infty, l^1)$ and the strict topology β. All these topologies on l^∞, as well as their respective duals are important mathematical objects on their own right and have been studied by influential mathematicians like Mackey (1943), Buck (1952, 1958), Conway (1965), and Collins (1968), among others. Of all the topologies, the strict topology has been most explored by inducing to its generalizations, see for instance, Shapiro (1971), Giles (1971), etc.

On the other hand, from the economic point of view any topology between weak and Mackey topologies captures a myopic behavior of individuals. Excellent references about this subject are the pioneer work of Bewley (1972) and the survey by Mas-Collel and Zame (1991). However, for an ample analysis on myopic topologies defined on l^∞, see Brown and Lewis (1981). Extensions to more general spaces are given by Raut (1986), and Stroyan (1983). For an extensive analysis of impatience (myopic) and its applications, see Araujo (1985) and Raut (1986).

To make things simply, this paper will only deal with intertemporal hyperopic behaviors in contrast to myopic behaviors studied by Brown and Lewis (1981) - who captured such behaviors via the Mackey topology.[2] More precisely, our concern in this paper will deal with the extreme behavior of agents in relation to the future. Our interest will be with agents who only see the distant future by completely neglecting any finite consumption stream. That is, the short run will be completely neglected. Since these agents are no longer myopic but rather hyperopic, the price system to be used by them will not be l^1 anymore. In their stead will be the set charges (or finitely additive measures). To reach such a characterization we use the concept of the Dunford-Schwartz integral where the integration is in relation to charges. For an ample study on this subject we recommend Rao and Rao (1983). A modern presentation of charges can be found in the recent and excellent reference due to Aliprantis and Border (2007).

The purpose of this paper is twofold. Firstly, we define another new topology on l^∞ and characterize its topological dual with respect to this new topology. We then show that this new topology captures a kind of intertemporal hyperopic behavior on the part of the individuals, and that its topological dual offers an appropriate way of pricing consumption paths belonging to l^∞. We also state that the elements of the hyperopic strict dual of l^∞ can be interpreted as linear utility functions for hyperopic agents.

We offer some examples of utility functions capturing hyperopic behavior. They involve limit sup and/or limit infimum of sequences representing infinity consumption paths as these kind of functions will only look at the tails of such streams by neglecting any finite stream of the whole stream. Since our aim

[2]In fact, any topology which is between the weak and Mackey topologies captures myopic behavior.

is purely topological, we do not make any axiomatic approach for representation of this kind of preferences. It is also useful to stress that our hyperopic agents have no inner conflict when deciding (as is characteristic of the self - control theory). They will only neglect any short losses or benefits and will only be interested in infinite streams (behavior on the infinite).

The paper is organized in the following way: Section 2 deals with hyperopic strict topologies which will allow us to formalize the concept of hyperopia via preferences. Section 2 ends by offering some examples of hyperopic preferences. In Section 3 we establish a representation theorem which allows us to prove that the hyperopic strict dual of l^∞ is the set of all pure charges. Section 4 contains some comments justifying hyperopic behaviors. This section ends by offering a literature related to the subject of this work. Finally, in Section 5, we give a short conclusion.

2 Hyperopic strict topologies

In this section we will guarantee the existence of a topology on l^∞ which will be called hyperopic strict topology. Hyperopic because it captures, as will be seen in Subsection 2.3, a kind of hyperopic behavior of individuals who have l^∞ as their consumption space. Strict because it is obtained from a a family of semi-norms indexed by a subset of l^1, and because it also recalls Buck (1958) who defined it for first time. Strict topology has been proven by Conway (1965) to be equivalent to the Mackey topology. Myopic behavior captured by these topologies was first noted by Hildebrand. See Bewley (1972) for a more ample discussion about myopic topologies.

Next, we define a family of seminorms which will induce our topology. It is its particular form of this family which will capture the hyperopic behavior of some individuals.

Let $\mathcal{A} \subset l^1$ be the set of all sequences $a \in l^1$ such that $a_n \neq 0$ for all but finitely many n. Thus, $\sum_{n \geq N} |a_n|$ is always non zero.

For each $a \in \mathcal{A}$, define on l^∞ the following function $p_a : l^\infty \to R_+$ to be

$$p_a(x) = \lim \sup_{N \to \infty} \frac{\sum_{n \geq N} |a_n x_n|}{\sum_{n \geq N} |a_n|}$$

Clearly, p_a satisfies the following properties:

1. Positivity
$$p_a(x) \geq 0$$

2. Homogeneity
$$p_a(\alpha x) = \lim \sup_{N \to \infty} \frac{\sum_{n \geq N} |a_n(\alpha x_n|)}{\sum_{n \geq N} |a_n|} = |\alpha| p_a(x)$$

3. Triangle Inequality
$$p_a(x+y) = \lim \sup_{N \to \infty} \frac{\sum_{n \geq N} |a_n(x_n + y_n)|}{\sum_{n \geq N} |a_n|} \leq p_a(x) + p_a(y)$$

(1) -(3) imply that p_a is a seminorm on l^∞.

2.1 Existence

The collection $\Gamma = \{p_a : a \in \mathcal{A}\}$ is a family of seminorms which defines a topology on l^∞ which in turn transforms l^∞ into a locally convex vector space.

For each $x \in l^\infty$, consider the sets $V_{A,\epsilon}(x)$ defined by

$$V_{A,\epsilon}(x) = \{y \in l^\infty : p_a(y - x) < \epsilon, a \in A\}$$

for every finite subset A of $\mathcal{A}$ and every $\epsilon > 0$.

Define $\mathcal{V} : l^\infty \to 2^{l^\infty}$ so that for each $x \in l^\infty$, $\mathcal{V}(x)$ consists of all sets $V \subset l^\infty$ such that there exists $V_{A,\epsilon}(x) \subset V$.

We have the following theorem.

Theorem 1. *Let $\mathcal{V}(x)$ be the collection of subsets defined above. Then the following holds:*

(1) The collection $\mathcal{V}(x)$ is a fundamental system of neighborhoods of $x \in l^\infty$.

(2) There is a unique topology β_h on l^∞ such that for each $x \in l^\infty$, $\mathcal{V}(x)$ is a fundamental system of neighborhoods of x. In addition, the members of β_h are characterized in the following way: $\mathcal{O}$ of l^∞ is open in τ if and only if $\mathcal{O} = \cup_{arbitrary} V_{A,\epsilon}(x)$.

(3) Let $\{x^n\}$ be a sequence in l^∞. Then $x^n \to x$ (in β_h) if and only if $p_a(x^n - x) \to 0, \forall a \in \mathcal{A}$.

(4) (l^∞, β_h) is a topological vector space (TVS) which is locally convex.

(5) $\beta_h \subset \tau_\infty$.

Proof. We will prove these items separately.

Proof of (1). Clearly $\mathcal{V}(x)$ satisfies the following properties: a) $x \in V$, for all $V \in \mathcal{V}(x)$; b)If $V \in \mathcal{V}(x)$ and $V \subset W$, then $W \in \mathcal{V}(x)$; c) If $V, W \in \mathcal{V}(x)$, then $V \cap W \in \mathcal{V}(x)$ and d) If $V \in \mathcal{V}(x)$, there exists $U \in \mathcal{V}(x)$ such that $y \in U$, then

$V \in \mathcal{V}(y)$. These conditions define a fundamental system of neighborhoods of x.

Proof of (2). This item follows from Proposition 2, §1, $n^{\underline{o}}2$ of Bourbaki (1965).

Proof of (3). For every $\mathcal{O} \in \tau$ containing x, there exists $n_o \in N$ such that $n > n_o$ implies that $x^n \in \mathcal{O}$. This in turn is equivalent to stating that $p_a(x^n - y) \to 0, \forall a \in \mathcal{A}$.

Proof of (4). The vector space operations are β_h-continuous. This follows from properties b) and c) of the definition of β_h. The local convexity of (l^∞, β_h) follows from the fact that each member, $V_{A,\epsilon}(x)$, of the generating family is convex.

Proof of (5). We will prove that any open set in τ_Γ contains an open ball in τ_∞. It is sufficient to prove that any $V(x, \epsilon, A) = \{y \in l^\infty : \rho_a(y - x) < \epsilon, a \in A\} \supset B_\infty(x, \epsilon) = \{y \in l^\infty : \sup_k |y_k - x_k| < \frac{\epsilon}{2}\}$. Let $y \in B_\infty(x, \epsilon)$. This implies that $|y_k - x_k| < \frac{\epsilon}{2}, \forall k$. Computing $\rho_a(y - x)$ with $a \in A \subset \mathcal{A} \subset l^1$ with A finite, one has

$$\rho_a(y-x) = \lim \sup_{N\to\infty} \frac{\sum_{n\geq N} |a_n(y_n - x_n)|}{\sum_{n\geq N} |a_n|} \leq \frac{\epsilon}{2} \lim \sup_{N\to\infty} \frac{\sum_{n\geq N} |a_n|}{\sum_{n\geq N} |a_n|}$$

therefore $\rho_a(y - x) < \epsilon$. □

2.2 Hyperopic preferences

In this section we are going to formalize the concept of intertemporal hyperopia. Intuitively, given any infinite stream, say, of consumption,[3] a hyperopic agent will neglect any finite stream of the stream, and will only see any tail of the whole stream. To be more precise, we are going to define the agents, who live forever, via their preferences which will represent their tastes on the set

[3]It may be a stream of wealth, capital, etc.

of all paths of consumption. They are assumed to be represented by bounded real sequences[4] belonging to the nonnegative cone, l^∞_+, of the space l^∞. This cone will represent the consumption set of a certain individual. Thus, one has the following definition:

Definition 1. *A preference relation for a certain individual is a subset $\succeq \subset l^\infty_+ \times l^\infty_+$ satisfying the following properties:*

1. *Reflexive:* $\forall x \in l^\infty_+, x \succeq x$
2. *Complete:* $\forall x, y \in l^\infty_+,$ *either* $x \succeq y$ *or* $y \succeq x$
3. *Transitive:* $\forall x, y, z \in l^\infty_+, x \succeq y, y \succeq z \Rightarrow x \succeq z$

By $x \succ y$ we mean that $x \succeq y$ and $\neg(y \succeq x)$, and $x \sim y$ means $x \succeq y$ and $y \succeq x$.

For any $x \in l^\infty$, we define its $n-$head denoted by x_{hn} to be

$$x_{hn}(k) = \begin{cases} x_k, & 1 \leq k \leq n \\ 0, & k > n \end{cases}$$

and its $n-$tail as $x^t_n = x - x_{hn}$.

Now we are ready to define hyperopic preferences as stated above in the beginning of this section.

Definition 2. *The preference relation $\succeq$ on l^∞_+ is said to be hyperopic if and only if it satisfies the following condition.*

$$\forall x, y, z \in l^\infty_+, \text{ if } x \succ y \text{ then } x \succ y + z_{hn}, \forall n. \tag{1}$$

(1) says that any increase z_{hn}, for a finite number of periods to the path of consumption y will not affect the relation $x \succ y$. That is, x will continue being preferred to $y + z_{hn}$ no matter

[4] Where each term of the sequence represents the consumption of the only good available.

when the provision z_{hn} begins. This last condition is too strong so that, by following the spirit of Brown and Lewis (1981), we weaken it by replacing "for all n" by "for all sufficiently large n."

Definition 3. *Let $\succeq$ be a preference relation on l_+^∞.*

1. *$\succeq$ is said to be weakly hyperopic $\Leftrightarrow$ for all $x, y \in l_+^\infty$ and c a constant sequence in l_+^∞, if $x \succ y$ then there exists N such that $x \succ y + c_{hn}$ for all $n \geq N$.*

2. *$\succeq$ is said to be strongly hyperopic $\Leftrightarrow$ for all $x, y, z \in l_+^\infty$ if $x \succ y$ then there exists N such that $x \succ y + z_{hn}$ for all $n \geq N$.*

3. *$\succeq$ is said to be monotonically hyperopic $\Leftrightarrow$ for all $x, y \in l_+^\infty$, if $x \succ y$ then there exists N such that $x_n^t \succ y_n^t$ for all $n \geq N$, where $x_n^t = x - x_{hn}$.*

Remarks 2.1: Hyperopia as defined by (1), clearly implies strong hyperopia which in turn implies weak hyperopia. However, note that if we assume some kind of classic monotonicity on the preferences, as for instance, $x_n \geq y_n, \forall n \Rightarrow x \succeq y$, then hyperopia, as defined above in (1), and Item 2 of Definition 1 are equivalent. In fact, strong hyperopia (Item 2 of Definition 2) implies that there exists N such that $x \succ y + z_{hn}, \forall n \geq N$. This fact together with the monotonicity just defined, imply that for all $n \leq N, x \succ y + z_{hN} \succeq y + z_{hn}$. Finally, strong hyperopia also implies monotonic hyperopia. In fact, suppose that $x \succ y$. Strong hyperopia implies that $x_{hn} \to_\tau 0$ and $y_{hn} \to_\tau 0$. It then follows that $x_n^t = x - x_{hn} \to_\tau x$ and $y_n^t = y - y_{hn} \to_\tau y$. Hence, there exists n_o, such that $x_n^t \succ y_n^t$ for all $n \geq n_o$.

The following definition is usual in the literature

Definition 4. *If a topology τ is given on l^∞_+, then a preference relation $\succeq$ is said to be $\tau-$continuous if $\succeq$ as subset of $l^\infty_+ \times l^\infty_+$ is closed.*

Remark 2.2: Since l^∞_+ is metrizable and second countable, continuity as defined in Definition 4 is the same as stating that for all $x \in l^\infty_+$ the sets $\{y \in l^\infty_+ : y \succeq x\}$ and $\{y \in l^\infty_+ : x \succeq y\}$ are closed in the topology τ.

2.3 Hyperopic topologies

Once the hyperopic preferences were defined we are ready to define hyperopic topologies. We offer three degrees of hyperopia:

Definition 5. *A topology τ on l^∞ is said to be weakly (strongly, monotonically) hyperopic if and only if every preference relation, $\succeq$, which is $\tau-$continuous, is weakly (strongly, monotonically) hyperopic.*

The following proposition shows that β_h is strongly hyperopic.

Proposition 1. *The hyperopic strict topology, β_h, is strongly hyperopic.*

Proof. We pick out any β_h-continuous $\succeq$ on l^∞ . Let $x, y, z \in l^\infty$. Suppose that $x \succ y$.

Computing $p_a(z_{hn})$ we have that $p_a(z_{hn}) = 0, \forall n$. This implies that

$$p_a(y + z_{hn} - y) = 0, \forall n, \tag{2}$$

and therefore

$$y + z_{hn} \rightarrow_{\beta_h} y \tag{3}$$

From the continuity of $\succeq$ one has that (3) implies that $x \succ y + z^n$, for any large enough n, implying strongly hyperopic $\succeq$. $\square$

Brown and Lewis (1981) have characterized myopic topologies via convergence to zero of the sequence of the tails of any bounded sequence. Here, we obtain a characterization for hyperopic topologies, but instead of using the sequence of tails we use the sequence of the heads of any bounded sequence.

Theorem 2. *If τ is a Hausdorff locally convex topology on l^∞ then τ is strongly hyperopic iff for all $z \in l^\infty$, $z_{hn} \to_\tau 0$*

Proof. In general lines the proof of this theorem follows the proof of an theorem in Brown and Lewis (1981) and may carry over to the present context with little changes. □

Remarks 2.3: To capture strong hyperopic behavior as defined in Item 2 of Definition 3 it is sufficient to assume upper semi-continuous preference relations with respect to τ. $\tau-$upper semicontinuity of $\succeq$ on l^∞_+ meaning that for each $x \in l^\infty_+$ the set $\{y \in l^\infty_+ : y \succeq x\}$ is closed[5] in the topology τ. We offer an example in the next section.

2.4 Example of hyperopic preferences

In this section we offer an example of strongly hyperopic preferences. More precisely, we give an example of upper semi-continuous preferences with respect to β_h. This will be a strongly hyperopic preference since individuals with this kind of preference will neglect any gains in finite paths. It will be shown above.

[5]Since the complement of the set $\{z \in l^\infty_+ : x \succ z\}$ is $\{y \in l^\infty_+ : y \succeq x\}$, it should be immediate that $\succeq$ on l^∞_+ is upper semicontinuous if for each $x \in l^\infty_+$ the set $\{z \in l^\infty_+ : x \succ z\}$ is open in the topology τ.

We consider preferences represented by the utility function $u : l_+^\infty \to R$ to be

$$u(x) = \limsup_{n\to\infty} x_n, \forall x \in l_+^\infty$$

Let $x^n \to_{\beta_h} y$. Then, $p_a(x^n - y) \to 0, \forall a \in \mathcal{A}$. This implies that there exists N_o such that for all $N \geq N_o$ one has

$$\frac{\sum_{k\geq N} |a_k(x_k^n - y_k)|}{\sum_{k\geq N} |a_k|} \leq \epsilon, \forall n \geq n_o$$

Therefore

$$|x_k^n - y_k| \leq \frac{||a||_1 \epsilon}{\inf\{a_k : a_k \neq 0\}}, \forall n \geq n_o$$

On the other hand, one has

$$u(x^n) - u(y) = \limsup_{k\to\infty} x_k^n - \limsup_{k\to\infty} y_k \leq \limsup_{k\to\infty} (x_k^n - y_k).$$

Then there exists a subsequence $(x_{k_m}^n - y_{k_m})$ of $(x_k^n - y_k)$ for which we have that

$$u(x^n) - u(y) \leq (x_{k_m}^n - y_{k_m}) < 2|x_{k_m}^n - y_{k_m}|, \forall m$$

Choose m_o such that $k_m \geq \max\{N_o, k_{m_o}\}$. Then,

$$2|x_{k_m}^n - y_{k_m}| \leq \frac{2||a||_1 \epsilon}{\inf\{a_{k_m} : a_{k_m} \neq 0\}}, \forall n \geq n_o$$

and therefore

$$u(x^n) - u(y) < \frac{2||a||_1 \epsilon}{\inf\{a_{k_m} : a_{k_m} \neq 0\}}, \forall n \geq n_o$$

Since ϵ is small enough, we have that

$$\limsup_{n\to\infty} u(x^n) \leq u(y)$$

and therefore it is upper semi-continuous. Suppose now that $u(x) > u(y)$. It is also known that $y + z_{hn} \rightarrow_{\beta_h} y$, because for every $a \in \mathcal{A}$ one has that $p_a(z_{hn}) = 0, \forall n$. But as u is upper semi-continuous with respect to β_h it follows that $u(x) > u(y + z_{hn}), \forall n$. This implies that gains in finite paths are neglected. Thus, the preference represented by the utility function above is strongly hyperopic.

Remarks 2.4: An argument similar to the one above shows that the preference relation represented by $u : l^{\infty}_{+} \rightarrow R$ defined by

$$u(x) = \liminf_{n\rightarrow\infty} x_n, \forall x \in l^{\infty}_{+}$$

is β_h-lower semi-continuous. That is, if $x^n \rightarrow_{\beta_h} y$, then $\liminf_{n\rightarrow\infty} u(x^n) \geq u(y)$. Suppose as above that $u(x) > u(y)$. From β_h- lower semi-continuity of u it follows that $u(x - z_{hn}) > u(y)$ for large enough n. This means that individuals with β_h- lower semi-continuous preferences will neglect losses in finite paths. Thus, they will also exhibit strongly hyperopic behavior. By drawing on the above arguments, we can infer that any function depending on $\limsup_{n\rightarrow\infty} x_n$ and $\liminf_{n\rightarrow\infty} x_n$ would exhibit some kind of strong hyperopic behavior, since for all $z \in l^{\infty}_{+}$, $z_{hn} = 0$, for all $k \geq n$, one has that for all $y, z \in l^{\infty}_{+}$ and for all n, one will have $\limsup_{n\rightarrow\infty}(y(k) + z_{hn}(k)) = \limsup_{n\rightarrow\infty} y(k)$. The same argument also holds for $\liminf_{n\rightarrow\infty} x_n$.

3 The dual of hyperopic topological spaces

Having characterized strong hyperopic agents who have the non-negative cone l^{∞}_{+} as a consumption set, the following step is to know how these agents value their consumption plans. The most natural thing to do is seek prices in the topological dual of l^∞ with respect to topology β_h. In order to characterize strong hyperopic price systems we will need a little of the theory of charges.[6] For an ample study on this subject we recommend Rao and Rao (1983). For the sake of completeness, we have listed, in the appendix, some definitions and results to be used in this section.

We begin by characterizing the β_h-continuous linear functionals. That is, $(l^\infty, \beta_h)'$.

Lemma 1. *A linear functional $f : (l^\infty, \beta_h) \to R$ is β_h-continuous if and only if $f(x_{hn}) \to 0$.*

Proof. This lemma follows from Proposition 1 and Theorem 2. In fact, β_h is strongly hyperopic. Therefore $x_{hn} \to_{\beta_h} 0$ for all $x \in l^\infty$. So, if f is continuous, then $f(x_{hn}) \to 0$. Conversely, if $f(x_{hn}) \to 0$ and f is not continuous, then the sequence $\{f(x_{hn})\}$ would not tend to zero which is a contradiction. Thus, Lemma 1 follows. □

It is well known that the dual, $(l^\infty, \tau_\infty)'$, is equivalent, see for instance Yosida and Hewitt (1952), to $ba(N)$ space which consists of the charges on the $\sigma-$field of all subsets of the natural numbers.

3.1 A representation theorem

Any pure charge (positive) $\mu \in ba$ induces a β_h-continuous linear functional. So, let $\mu \in \mathcal{C}_p(N)$ be a pure charge. Define the following linear functional $H_\mu : (l^\infty, \tau_\Gamma) \to R$ to be

$$H_\mu(x) = D \int_N x(n) d\mu, \forall x \in l^\infty$$

[6] or finitely additive measures.

where the integral on the right hand is known as the Dunford-Schwartz integral.

For any $x \in l^\infty$ the sequence of $n-$heads of x is defined to be

$$x_{hn}(m) = x(m), 1 \leq m \leq n, \text{ and } x_{hn}(m) = 0, m > n.$$

Thus

$$H_\mu(x_{hn}) = D\int_{N_n} x(n)d\mu \leq ||x||_\infty \mu(N_n), \text{ where } N_n = \{1, \ldots, n\}$$

From Theorem 3.2 and Theorem 5.7 (pages: 189 and 193 respectively) in Olubummo (1969), it follows that $\mu(N_n) = 0$. Therefore $H_\mu(x_{hn}) = 0$ since μ is positive (see Definition 6 in the appendix). Therefore, by using Lemma 1 above, it follows that H_μ is β_h-continuous.

Theorem 3. *For all $F \in (l^\infty, \beta_h)'$, there exists a unique bounded pure charge, μ, such that*

$$F(x) = D\int_N x(n)d\mu$$

Proof. From Item 5 of Theorem 1 it follows that $f \in (l^\infty, \tau_\infty)' = ba(N)$. From Theorem 4.7.4 in Rao and Rao (1983), one has that there exists a unique bounded charge $\mu \in ba(N)$ such that:

$$F(x) = D\int_N x(n)d\mu(n), \forall x \in l^\infty.$$

where the integral on the right hand is known as the Dunford-Schwartz integral.

From Theorem 1.23 in Yosida and Hewitt (1952) there exists a unique decomposition for $\mu = \mu_c + \mu_p$ with μ_c countably additive and μ_p purely finitely additive. Thus

$$F(x) = D\int_N x(n)d\mu_c(n) + D\int_N x(n)d\mu_p(n)$$

Since $ca(N) \equiv l^1$, we have that there exists $a \in l^1$ defined by $a_n = \mu_c(n), \forall n$, such that the first integral can be written as a series. Thus F can be rewritten as:

$$F(x) = \sum_n^\infty x_n a_n + D \int_N x d\mu_p(n)$$

It is easy to show that the only continuous linear functional $F_{\mu_c}(x) = \sum_n^\infty x_n a_n$ with respect to topology β_h is zero. Thus $\mu_c = 0$.

So

$$F(x) = D \int_N x(n) d\mu_p(n)$$

Thus, we have shown that for any F defined on l^∞ which is linear and β_h-continuous, there is a unique pure charge μ which represents F. □

The argument at the beginning of Section 3.1 and Theorem 3 imply the following corollary.

Corollary 1.

$$(l^\infty, \beta_h)' = \{\mu \in ba(N) : \mu \textit{ is a bounded pure charge}\}$$

Remark 6.1: The proof of Theorem 3 corresponds to the case of positive pure charges. For the general case it is possible to invoke the Jordan decomposition.

4 Related Literature

The choice of infinite-dimensional spaces to represent consumption spaces has been justified by several authors for a long time, see Bewley (1972); and Mas-Collel and Zame (1991). In these works no generalization of the notion of a price system was required. They continued being considered lists. But since the topological dual of any infinity dimensional space depends on the topology defined on it, and since, on this kind of spaces, there are generally many topologies, a good choice of topology must be made in order to preserve the price system. For instance, when it is required that l^1 model the price system, the best topology to be chosen is Mackey, which is the topology of the convergence on compact sets with respect to the weak topology of l^∞. Therefore, a natural assumption on preferences of agents is that they are Mackey continuous.

The interest of studying preferences which are not Mackey continuous in the context of general equilibrium goes back to Bewley (1972). He, as well as Werner (1997) and Giles and LeRoy (1992), obtained equilibria in economies with such preferences as those we have just named, with prices in the space of charges[7]. Bewley (1972) gives conditions to eliminate the purely finitely additive part of the equilibrium pricing functional, since at that time he thought that the non-additive part of such a functional makes no economical sense. However, Werner (1997) and Giles and LeRoy (1992) gave it an economic interpretation, namely, pricing bubbles. Lastly, the possibility of pricing bubbles under Mackey upper but not lower semicontinuous preferences was dealt with by Araujo, Novinski and Pascoa (2011). To the best of our knowledge, none of the previous papers have offered topologies capturing the behavior at infinity, except that of Gilles (1989).

On the other hand, the use of charges in economics is not new. They have been used by several authors. Among them

[7]finitely additive measures

we find Werner (1997), who in two occasions deals with diversification and valuation in securities markets. Gilles and LeRoy (1992) formalize, via charges, the standard definition of rational speculative bubbles. An excellent survey on asset price bubbles was recently offered by Jarrow, Protter and Shimbo (2010). In addition to characterizing bubbles, charges are also used to model catastrophic risks, see for instance Chichilnisky (1996).

5 Conclusion

We have constructed a new topology on l^∞ which we have been called hyperopic strict topology, because it captures hyperopic behavior of individuals. This topology, although having been defined by a family of semi norms, yields a topological dual which is no longer l^1 but rather the set of pure charges. We interpret each element of this dual to be continuous linear utility function on l^∞, which would be the counterpart of discounted utilities for myopic agents. But this relevant topic, as well as the identification of axioms required for this representation, would be subjects for future research.

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Appendix

Proposition 2. *Let Γ be a family of semi norms in a vectorial space V. The topological space (V, τ_Γ) is Hausdorff if and only if for each $x \in V$ not zero, $\exists p \in \Gamma$ such that $p(x) \neq 0$.*

Definition 6. *Let $\mathcal{F}$ be a field of subsets of a set Ω. A map $\mu : \mathcal{F} \to [-\infty, +\infty]$ is said to be a charge on $\mathcal{F}$ if the following conditions are satisfied:*

1. $\mu(\phi) = 0$

2. $\mu(A \cup B) = \mu(A) + \mu(B), \forall A, B \in \mathcal{F},$ *with* $A \cap B = \phi$

A charge on $\mathcal{F}$ is known in the literature as a finitely additive measure. For the sake of completeness and comparison, next we define the concept of a measure.

Definition 7. *Let $\mathcal{F}$ be a field of subsets of a set Ω. A measure on $\mathcal{F}$ is any map $\mu : \mathcal{F} \to [-\infty, +\infty]$ having the following properties.*

1. $\mu(\phi) = 0$

2. *If $\{F_n\}_{n\geq 1}$ is a sequence of pairwise disjoints sets in $\mathcal{F}$ with $U_{n\geq 1} F_n \in \mathcal{F}$, then*

$$\mu(U_{n\geq 1} F_n) = \sum_{n\geq 1} \mu(F_n).$$

Any measure, obviously, is a charge. The converse is false. A measure (respectively a charge) μ is said to be bounded on $\mathcal{F}$ if

$$\sup_{F\in\mathcal{F}} \mu(F) < +\infty$$

A measure is known in the literature as a countably additive measure. The set of bounded charges (resp. measures) is denoted by $ba(\Omega, \mathcal{F})$ and $ca(\Omega, \mathcal{F})$ respectively. The sets $ba_+(\Omega, \mathcal{F})$ and $ca_+(\Omega, \mathcal{F})$ are, respectively, the sets of all positive charges and measures.

Definition 8. *A positive charge μ on $\mathcal{F}$ is called a pure charge if*

$$\neg(\exists \lambda \in ca_+(\Omega, \mathcal{F}) : \lambda \leq \mu)$$

In other words,

$$\mu \text{ is a pure charge } \Leftrightarrow \forall \lambda \in ca_+(\Omega, \mathcal{F}); 0 \leq \lambda \leq \mu \Rightarrow \lambda = 0$$

D-integrability

Definition 9. *Let* $(\Omega, \mathcal{F}, \mu)$ *be a charge space. The function* $f : \Omega \to R$ *is said to* $D-$*integrable if* $\exists \{f_n\}_{n \geq 1}$ $D-$*integrable simple functions such that*

1. f_n, *converge to* f *hazily*[8] *as* $n \to \infty$

2. $$\lim_{m,n \to \infty} D \int |f_n - f_m| d|\mu| = 0.$$

If f *is* $D-$ *integrable, the* $D-$*integral of* f *is denoted by* $D \int f\mu$ *and is defined to be the number* $\lim_{n \to \infty} D \int f_n d\mu$. *That is,*

$$D \int f\mu = \lim_{n \to \infty} D \int f_n d\mu.$$

[8] f_n is said to converge to f hazily if $\lim_{n \to \infty} |\mu|^*(\{\omega \in \Omega : |f_n(\omega) - f(\omega)| > \epsilon\}) = 0$.

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