Welfare and Taxation in an Asset Pricing Model with Dispersed Information

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Introduction

- In an influential work, Fama (1970) presented the different forms of the *efficient market hypothesis* and reviewed the existing theoretical and empirical work on the subject.
- Since then there has been a vast empirical literature testing market efficiency.
- Also, many theoretical models were also developed to explain price as a conveyor of information.

Introduction

- Grossman and Stiglitz (1980): showed impossibility of efficient markets when information is costly to acquire.
- Hellwig (1980) and Diamond and Veracchia (1981) analyzed the equilibrium in a large market model with noise trading. Their equilibrium did not feature *schezophrenic traders* as in Grossman and Stiglitz (1980).
- Several papers on analyzing different variations of asset pricing models with dispersed information, but few looking at the effects of policies from a theoretical point of view.

- There is also a literature looking at price as a mechanism to aggregate information outside the financial markets scenario.
- Vives (1988) analyzed how information is aggregated through prices in large Cournot markets where firms had private information. Showed competitive market is efficient given restrictions imposed by decentralized information.
- Messner and Vives (2001) looked at possible gap between information and economic efficiency in a rational expectations competitive market with dispersed information.
- Vives (2014a) studied market games with endogenous public information and pointed out the existence of two different externalities in the use of information: learning and pecuniary.

Related Literature

- Literature on noisy information aggregation in asset pricing models.
  - Green (1973), Grossman (1976, 1978), Radner (1979), Grossman and Stiglitz (1980), Hellwig (1980), Diamond and Veracchia (1981), Albagli, Hellwig, and Tsyvinski (2013), Vives (2014), among others.
- Literature on welfare in noisy rational expectation equilibrium.
  - Vives (1993) Vives (1988), Amador and Weil (2012), Banerjee (1992), Vives (2014), and others.

Related Literature

- After financial crisis, there has been a growing interest in studying the effects of taxation in financial markets.
- Most of the work has been empirical and theoretical ones usually do not focus on models with dispersed information and closed form solutions.
  - Shi and Xu (2008) - look at Tobin tax in model of foreign exchange market with noise trading. Show that tax may increase exchange rate volatility when entry decisions of traders are endogenized.
  - Rieger (2014) - look at the effects of financial transaction tax on trading volume and asset price volatility in model with heterogenous beliefs. Mixed results regarding traded volume, volatility, and welfare.

- The motivation for this paper came because of the debate that was triggered by the recent financial crisis. The center of this debate need for and effectiveness of interventions in capital markets
- There has not been a concensus on the subject and the empirical evidence and theoretical work show mixed results.

Goal of this Paper

- Contribute to theoretical literature on the effects of tax policy in capital markets in which information is dispersed.
- Analyze the equilibrium of an asset pricing model when there is a tax on returns.
- See how the tax may be used to affect:
  1. the role of price as an aggregator of information.
  2. welfare - as measured by the *team solution*
  3. volality of returns
  4. other measures of market efficiency

Payoff Environment

- Payoff environment similar to the ones studied in Hellwig (1980) and Diamond and Veracchia (1981).
- Focus on large markets.
- Continuum of players of measure one. An individual trader is indexed by $i \in [0, 1]$.
- There is a risky asset whose value is a realization of $\theta \sim N(\mu, \frac{1}{\tau_\theta})$.
- Asset is sold at price $P \in \mathbb{R}$.

Payoff Environment

- Each trader chooses the number of shares, $S_i \in \mathbb{R}$, of the risky asset that they want to buy.
- Agent i's utility is given by:

$$U(S_i, \theta) = \mathbb{E}[(\theta - P)S_i] - \frac{\rho}{2} Var[(\theta - P)S_i]$$

- The parameter ρ captures the agent's dislike of risk.

Belief Environment

- Traders have the same prior on the realization of the asset value: $\theta \sim N\left(\mu, \frac{1}{\tau_\theta}\right)$
- Price, P, is public information at the time of the trade.
- Traders get a private signal, $x_i = \theta + \varepsilon_i$, with $\varepsilon_i \sim N\left(0, \frac{1}{\tau_x}\right)$ i.i.d.
- Distributional assumption: $\int_0^1 \varepsilon_i d_i = 0$.

Belief Environment

- There are noise traders with aggregate demand $u \sim N\left(0, \frac{1}{\tau_u}\right)$, i.i.d.
- Agent i's information set is $G_i = (x_i, P)$ and so we denote trader i's strategy as $S_i(x_i, P)$.
- Trader i chooses S_i to maximize

$$U_i(S_i, \theta | G_i) = \mathbb{E}[(\theta - P)S_i | x_i, P] - \frac{\rho}{2} Var[(\theta - P)S_i | x_i, P]$$

Equilibrium Concept

Definition 1 (Symmetric Rational Expectations Equilibrium)

A symmetric rational expectations equilibrium is a set of trades, $S(x_i, P)$, and a measurable price functional $P(\theta, u)$ such that:

1. *Trader i is optimizing:*

$$S(x_i, P) \in argmax_z \, \mathbb{E}[(\theta - P)z] - \frac{\rho}{2} Var[(\theta - P)z] \; \forall i \in [0, 1].$$

2. *Market clears:*

$$\int_0^1 S(x_i, P) d_i + u = 0.$$

Equilibrium Concept

- We look at how this type of equilibrium can be implemented as a Bayesian equilibrium in demand functions.
- Agent's when optimizing take into account the relationship between P and the random variables (θ, u) that is established through market clearing.
- Because of the form of utility function and Gaussian uncertainty we will actually have *linear* Bayesian equilibria.

Finding the Equilibrium

We take the following steps in order to characterize the equilibrium:

- Conjecture linear strategies for the traders:

$$S(x_i, P) = a + cx_i - bP$$

and, using market clearing, obtain

$$P(\theta, u) = \frac{a + c\theta + u}{b}$$

Finding the Equilibrium

- Update traders beliefs using the expression found in the previous step. That is, find:

$$\mathbb{E}[\theta|x_i, P]$$

- Calculate demand for the traders considering their utility maximizing behavior.
- Identify coefficients of linear demands by imposing consistency between conjectured and actual strategies.

Equilibrium

Proposition 1 (Linear Bayesian Equilibrium in Demand Functions)

There is a unique linear Bayesian equilibrium in demand functions and it is given by:

$$S(x_i, P) = \frac{\tau_\theta \mu}{\rho + \tau_x \rho^{-1} \tau_u} + \frac{\tau_x}{\rho} x_i - \frac{\tau_x + \tau_\theta + \tau_x^2 \rho^{-2} \tau_u}{\rho + \tau_x \rho^{-1} \tau_u} P$$

$$P = \frac{\tau_\theta \mu}{\tau_\theta + \tau_x + \tau_x^2 \rho^{-2} \tau_u} + \frac{\tau_x + \tau_x^2 \rho^{-2} \tau_u}{\tau_\theta + \tau_x + \tau_x^2 \rho^{-2} \tau_u} \theta + \frac{\rho + \tau_x \rho^{-1} \tau_u}{\tau_\theta + \tau_x + \tau_x^2 \rho^{-2} \tau_u} u.$$

Equilibrium

- Proposition 1 is similar to Proposition 5.2 in Hellwig (1980) and to the equilibrium characterization in Diamond and Veracchia (1981).
- Diamond and Veracchia (1981), however, consider a finite number of agents and assume random endowment of the risky asset instead of noise trading.
- Hellwig (1980) considers a continuum of agents and noise traders, but he assumes each trader has a different degree of risk aversion. Or, in the context of our model, a different ρ_i for each trader.

Welfare

- Before introducing taxation we need to first determine the welfare criterion and the possible sources of inefficiencies in this model.
- Unlike traditional planner's problem, here we cannot maximize over allocations while disregarding prices.
- Need to take into account restrictions on how information is communicated.
- We choose to look at the *team solution*. Concept has been used in the literature. See Radner (1979), Vives (1988), Angeletos and Pavan (2007), and Vives (2014a).

Team Solution

- We choose not to include expected utility of noise traders in our expression for welfare.

Definition 2 (Planner's Problem)

The planner choose coefficients (a, b, c) to maximize the aggregate ex-ante utility of traders given the dispersed nature of information. That is,

$$max_{(a,b,c)} \left[\int_0^1 \mathbb{E}[(\theta - P)S_i] - \frac{\rho}{2} Var[(\theta - P)S_i] \right]$$

subject to $S_i = a + cx_i - bP$, $S = a + c\theta - bP$, and $S + u = 0$.

Sources of Inefficiencies

- There are three sources of inefficiencies in our competitive REE.
- Learning externality: agents learn from price, but don't take into account how they affect P as a signal of θ

$$Var[\theta|P] = (\tau_\theta + \tau_u c^2)^{-1}$$

- Payoff externality: agent's don't take into account how they affect volalitity of price through their effect on aggregate demand.
- Because agents maximize at an interim stage, they don't account for variance of the conditional expectation.

Tax on Returns

- Here we consider a tax on returns. That is, capital gains are taxed and, symmetrically, capital losses will be attenuated by this intervention.

- Also allow planner to balance budget by using a lump sum transfer. With this tax policy the trader's payoff becomes:

$$R(t, t_0) = (\theta - P)(1 - t)S_i + t_0$$

in which $t \in [0, 1]$ is the rate at which returns are taxed and $t_0 \in \mathbb{R}$ is the lump sum transfer.

Equilibrium with Tax

Proposition 3 (Linear Bayesian Equilibrium in Demand Functions with a Tax on Returns)

There is a unique linear Bayesian equilibrium in demand functions when $t \in [0, 1)$, and it is given by:

$$S(x_i, P) = \frac{\tau_\theta \mu - [\tau_x + \tau_\theta + \tau_x^2 \rho^{-2}(1-t)^{-2}\tau_u]P}{\rho(1-t) + \tau_x \rho^{-1}(1-t)^{-1}\tau_u} + \frac{\tau_x}{\rho(1-t)} x_i$$

$$P = \frac{\tau_\theta \mu + [\tau_x + \tau_x^2 \rho^{-2}(1-t)^{-2}\tau_u]\theta + [\rho(1-t) + \tau_x \rho^{-1}(1-t)^{-1}\tau_u]u}{\tau_\theta + \tau_x + \tau_x^2 \rho^{-2}(1-t)^{-2}\tau_u}$$

Relative Importance of Private Information

- Ratio of weights given to x_i and P, respectively.

$$\frac{c}{b} = \frac{\tau_x + \tau_x^2 \rho^{-2}(1-t)^{-2}\tau_u}{\tau_x + \tau_x^2 \rho^{-2}(1-t)^{-2}\tau_u + \tau_\theta}.$$

- Ratio is increasing in the tax rate, t.
- Traders feel more confident in speculating on their private information.

Market Depth

- Market depth captures how much the market can absorb a shock in noise trading without changing prices.

$$MD = \frac{\tau_\theta + \tau_x + \rho^{-2}(1-t)^{-2}\tau_x^2\tau_u}{\rho(1-t) + \tau_x\rho^{-1}(1-t)^{-1}\tau_u}$$

- Equals the average responsiveness of traders to market price.
- Effect of t is ambiguous. To understand note that we can rewrite the expression ofr market depth as:

$$MD = \frac{Var(\theta)}{[Var(\theta) - Var(\theta|x_i, P)]Var(x_i|\theta)\rho(1-t)}.$$

Market Depth

- An increase in t has two effects:
  - It has a direct positive effect on market depth through the $(1 - t)$ showing up in the denominator. This captures the fact that tax has a direct negative effect on the volatility of returns.
  - It has a negative effect because it increases $Var(\theta) - Var(\theta | x_i, P)$.
- Necessary condition for MD to be decreasing in t is $\tau_\theta > 2\tau_x$.

Precision of Price in the Estimation of θ

- The informational content of prices is often measured by its precision in the estimation of θ. In our model this is given by:

$$\frac{1}{Var[\theta|P]} = \tau_\theta + \tau_u \tau_x^2 \rho^2 (1-t)^{-2}$$

- It is increasing in the tax rate, t.
- A larger tax rate makes the trader's more confident in speculating on their private information, thus, increasing the weight on private information.
- This makes price more correlated with θ. That is, price becomes a better conveyor of value.

Motivation for Trading

- The equilibrium strategy may be rewritten as:

$$S(x_i, P) = \frac{1}{(1-t)\rho Var[x_i|\theta]}[x_i - P] + \frac{1}{(1-t)\rho Var[\theta|P]}\left[\mathbb{E}[\theta|P] - P\right]$$

- **Speculative Trading:** exploits informational advantage. Increasing in t.

- **Market Making:** exploits possible discrepancies between price and public information about the fundamental. Effect of an increase in t is ambiguous.

Optimal Policy with $t_0 = 0$

- We first consider the problem when the planner does not use the lump sum transfer.

  In this scenario, the planner's problem is

$$\max_{t\in[0,1]} \int_0^1 \left(\mathbb{E}[(\theta - P)S_i(1-t)] - \frac{\rho}{2} Var[(\theta - P)S_i(1-t)]\right) di,$$

  subject to S_i and P being part of the Bayesian equilibrium in demand functions.

Optimal Policy with $t_0 = 0$

Proposition 4 (Optimal Tax on Returns when $t_0 = 0$)

When the planner sets the lump sum transfer equal to zero, then the policy that solves the planner's problem takes the following form

$$t^* = \begin{cases} 0 \text{ if } \tau_u < \frac{4\rho^2}{\tau_x} \text{ and } \tau_\theta \geq \frac{4\rho^4 + 3\tau_u\tau_x\rho^2 - \tau_u^2\tau_x^2}{\tau_u\rho^2} \text{ or } \tau_u > \frac{4\rho^2}{\tau_x} \\ \in (0,1) \text{ if } \tau_u < \frac{4\rho^2}{\tau_x} \text{ and } \tau_\theta < \frac{4\rho^4 + 3\tau_u\tau_x\rho^2 - \tau_u^2\tau_x^2}{\tau_u\rho^2} \end{cases}$$

Proposition 5 (Noise Trading and Intervention)

1. $\forall \tau_u > \frac{4\rho^2}{\tau_x}$ the optimal tax rate is $t^* = 0$.
2. *When $t^* > 0$, then it is increasing in the amount of noise trading (τ_u^{-1}).*

Optimal Policy with $t_0 \neq 0$

- Next, we consider a situation in which the planner can use a lump sum transfer to achieve expected budget balance.
- In our model, we can show that this lump sum transfer would be positive.
- The planner's problem in this situation would be:

$$max_{t\in[0,1]} \int_0^1 \left(\mathbb{E}[(\theta - P)S_i] - \frac{\rho}{2} Var[(\theta - P)S_i(1-t)] \right) di,$$

subject to S_i and P being part of the Bayesian equilibrium in demand functions.

Optimal Policy with $t_0 = 0$

Proposition 6 (Strictly Positive Tax is Optimal)

If the planner sets $t_0 = \mathbb{E}[t(\theta - P)S_i]$*, then* $t^* \in (0, 1)$*.*

- Result shows that when the *direct* effect of the tax on expected return is neutralized by a lump sum transfer, then the planner can always find a strictly positve tax rate that improves upon the competitive outcome.
- Shutting down the market, i.e. $t = 0$ is never optimal.

Unconditional Volatility

- In foreign exchange markets, authorities usually care about the volatility of the unconditional variation in the exchange rate.
- We have a static model, but the model could be adapted and interpreted in such a way that $Var[\theta - P]$ would be a proxy for this type of measure.

Proposition 7 (Unconditional Volatility)

The variance of the difference between price and fundamental, i.e. $Var[\theta - P]$*, is strictly decreasing in* t *for* $t \in (0, 1)$.

Semi-Strong Efficiency

- Many times the price is the focal point for measuring market efficiency.
- The different forms or the efficient market hypothesis, for example, look at what kind of information the price incorporates.
- Given the information structure of our model we choose to analyze the *semi-strong efficiency*, or, whether price reflects all public information.
- The price is semi-strong efficient if and only if $|\mathbb{E}[\theta|P] - P| = 0$

Semi-Strong Efficiency

- We have that in our model the price is not generically semi-strong efficient:

$$|\mathbb{E}[\theta|P] - P| = \frac{\tau_\theta (1-t)^4 \rho^4}{(\tau_u \tau_x + (1-t)^2 \rho^2)(\tau_u \tau_x^2 + \tau_\theta (1-t)^2 \rho^2} |(\mu - P)|$$

- An increase in the tax rate does reduce this form of inefficiency, but it may not eliminate it.

- We next consider a transaction tax $T_i = (t_1 + t_2 P) S_i$.

Transaction Tax

- Potential problem with this tax: it depends on just S_i and so if this was interpreted in the context of firms we could have an issue with the firm's decision about capital structuring.

- We abstract from these issues for now.

Proposition 8 (Semi-Strong Efficiency)

If taxes are set such that

$$t_1^* = \frac{-\mu\tau_\theta\rho^4}{\tau_u\tau_x(\rho^2(\tau_\theta + \tau_x) + \tau_u\tau_x^2)}$$

$$t_2^* = \frac{\tau_\theta\rho^4}{\tau_u\tau_x(\rho^2(\tau_\theta + \tau_x) + \tau_u\tau_x^2)}$$

then the price will be semi-strong informationally efficient.

Transaction Tax

- Even though this type of tax makes price semi-strong efficient, thus, eliminating the possibility of agents *leaning against the wind*, it does not effect the payoff after tax payments.
- So this type of tax does not affect welfare. However, it this policy runs an expected deficit, i.e. $\mathbb{E}[(t_1^* + t_2^* P)S_i] < 0$.
- So it the planner needed to use lump sum tax to recover losses, then there would be a trade-off between welfare (as measured by the team solution) and semi-strong efficiency of prices.
- This tax does not change the equilibrium weight attributed to private information, c, and so it does not effect the information content of price.

Concluding Remarks

- Looked at taxation in a single asset pricing model and analyzed implications of this policy to the equilibrium.
- Focused on how a tax on returns could be used to improve welfare under certain market conditions.
- Tax on returns also increased the precision of price in the estimation of the fundamental and reduced unconditional volatility of the gap between fundamental and price.
- A transaction tax could be used to attain semi-strong efficiency of prices and eliminate market making opportunities.

Concluding Remarks

- We believe that the results from this work provides us with some possible topics for future research.
- First is to look more carefully at the different sources of inefficiency and being able to disentagle them in our model.
- We also consider looking at a variation of this model that endogenizes the shocks to market liquidity. Possibility is to look at policy in a model similar to Mendrano and Vives (2007).

Concluding Remarks

- A third possibility is condering a dynamic version of the current model and understand how taxes could affect the learning from prices.
- Finally, since we see a lot of interventions by Central Banks in the foreign exchange markets, it would be interesting to do a similar analysis to a model that captured the specific aspects of that market.