Tests for history dependence in mixed-Poisson growth: Brazil, 1822-2000, and USA, 1869-1996, with an estimate of the world mixing distribution at start-up

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Tests for history dependence in mixed-Poisson growth: Brazil, 1822-2000, and USA, 1869-1996, with an estimate of the world mixing distribution at start-up

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Abstract

The growth empirics of two separate but related issues are studied. In the first, the annual data on GDP per person (GDPpp) for Brazil, 1822-2000, and the US, 1869-1996, were converted into arrival times for innovations defined as permanent increments to GDPpp of given size (e.g. 3%). We say an economy exhibits history dependence if its arrival times can be shown to come from a homogenous Poisson process, that is, with a constant mean arrival rate of innovations (the Poisson parameter, λ). Brazil’s growth trajectory since 1822 is shown not to be history dependent in this sense, though both its truncated series, 1889-2000, and the US, 1869-1996, passed the tests. Brazil’s stagnation in the mid to late 19th century, when coupled to its growth spurt in the 20th, is the suggested reason for the failure. The second study uses the Summers-Heston data for 134 economies in 2000 to estimate a discrete, theoretical distribution for the λ at start-up in 1800, based on the (unproved) assumption that each economy drew an arrival rate from this “mixing” distribution and stayed with it. The results show that if innovations are assumed to be big, rare events, then the world mixing distribution will be skewed, and all the λ drawn at start-up will be almost equal and thus the dispersion of incomes across nations 200 years later will then be due only to random variations along paths generated by the same homogenous Poisson process.

Key words: stochastic growth, mixed Poisson processes, statistical tests
JEL classes: O41, C15

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Tests for history dependence in mixed-Poisson growth: Brazil, 1822-2000, and USA, 1869-1996, with an estimate of the world’s mixing distribution in 1800

In Aghion and Howitt [1992], the source of growth in any economy is some kind of Poisson stochastic process for the occurrence of innovations, each of which increases instantaneously and permanently the GDP per person by an exogenous percentage. De Castro and Gonçalves [2001, 2003] showed how to use this notion to interpret as a world income distribution, the innovation counts from a set of initially identical economies that were made to follow a theoretical, mixed Poisson process since start-up, say 1800. The most general such process for a single economy is one in which the Poisson parameter ($\lambda$), the mean arrival rate of innovations, varies with both time and the accumulated number of innovations. Here we use the inverse of this interpretation to convert the annual time series of GDP per person for the Brazilian and US economies into arrival times of innovations, and tested whether each sequence of arrivals could have been generated by a homogenous Poisson process, namely with constant $\lambda$. If its trajectory passes these tests, then we say the economy’s growth is history dependent, in the sense that at start-up it drew a $\lambda$ and stayed with it.

Both rich and poor countries may exhibit this type of path. This is because the tests are not concerned with the average number of innovations over the whole trajectory, the parameter in the theory that would largely generate rich and poor countries at the end of the 20th century, but rather whether this average number was sustained or not. If an economy should have had intervals of sustained deviations from its average, equivalent to both long periods of stagnation and growth spurts, then it would not pass the tests for history dependence.

Brazil, 1822-2000, did not pass the tests but both its truncated series for 1889-2000, as well as the one for the US, 1869-1996, did. Graphs 1 and 3 in the appendix illustrate the difference in the trajectories for the two economies. Brazil’s long period of stagnation in the second half of the 19th century, because it was followed by a growth spurt in most of the 20th, is the cause of the rejection of the history dependence hypothesis. From 1835, it took nearly 87 years to increase GDP per person by 3% (table 1 in appendix), represented by the nearly vertical line between the 14th and 15th innovation, each defined as of “size” 3% in the graphs (see Section 1.1). The US in contrast (graph 3) was never far from its average growth rate, meaning that its innovations of “size” 3% continued to arrive at a steady pace over its trajectory. By the same token we expect countries like India and China that are poor in 2000 may also pass the test because their low average rates of growth, and thus low average rates of arrival of innovations, must have been sustained throughout. Recently, both have entered a period of high growth which, if continued for long enough, may cause the hypothesis eventually to be rejected.

Even if it exhibits history dependent growth, a country can be poor today either because it drew a low-valued $\lambda$ at start-up, or because it had a bad outcome from a high-valued one. The latter is not a low probability event if one remembers the shape of the Poisson distribution, which would be the theoretical distribution for the number of innovations since start-up. In Section 5.4 we report our estimated theoretical, world “mixing” distribution from which each economy purportedly drew its $\lambda$, using the (unproven) assumption that all economies’ growth paths were history dependent. As we increased the definition of “size” of innovation, varying it from 1 to 7%, the “mixing” distribution became more negatively skewed, that is, the higher values of $\lambda$ became more likely. Put another way, if the view of narrative economic history should be that a small number of isolated, large innovations has been the main engine of growth, then very few economies would have drawn a low $\lambda$ and the main cause of the dispersion 150 years later would
be outcomes from nearly equal mean arrival rates of innovations in the same stationary process shared by almost all economies.

One justification for this line of growth research which removes completely the focus on business cycles, comes from a recent paper which studied them in the Brazil trajectory, 1850-2000 (Araújo et al [2008]). These authors found that volatility in its cycles was not significantly correlated with its growth rates, leading them to conclude that “(any) policy aimed at smoothing out the business cycles is unlikely to foster long-run growth (p.577)”.

The rest of the paper is laid out as follows. The next sub-section explains the derivation of the occurrence times of innovations from the GDP per person series. The second section gives some economic theoretic background. The third specifies the statistical methods used and shows their connection with orthodox, time-series econometrics which tests for random walks directly, the fourth gives the sources of the data and the final section outlines the main results.

1.1 The conversion from GDP per person to innovations

An innovation is assumed to be the cause of any permanent increment in GDP per person. From any given point in time, say start-up sometime around 1800-50, if we observe a subsequent, fixed, percentage increase in GDP per person, we say that another innovation had occurred when this increase is attained. It seems that until then, all economies had more or less the same income per person due mainly to the absence of innovations. Unlike Lucas [2002] who argues that innovations did occur but were consumed by population growth, we suggest that since there was little population growth before the first industrial revolution, the observed stagnation implied no innovations. Like Lucas though, we accept Bairoch’s [1993] correction of Maddison’s tendency to exaggerate the early 19th century differences between the future first and third worlds. His more recent publications have tried to reduce this bias (see Maddison [1995]). Some economies have such low incomes in 2000 (see table 6) that no innovations, both in common parlance and our specific sense, would seem to have occurred in them over the 150 or so years. Start-up therefore is not necessarily take-off. A few economies started to grow immediately while most continued to stagnate, some with significant population growth.

The basic assumptions made are one, that the only source of permanent increments to GDP per person are random events called innovations and two, that these come in the form of a fixed percentage increase which we vary in some experiments between 1% and 7%. Any subsequent decline is not treated as retrogression and is thus ignored. So an event, the arrival of an innovation, occurs each time the trajectory completes an increment of a given percentage in the GDP per person. To illustrate, if the observed series exhibited exponential growth at a constant rate, the imputed intervals between arrivals derived from the conversion will have a constant value.

The arrival times for innovations, $T_i$, were obtained from the following procedure. Starting from the base year, 1822 for Brazil, 1869 for the US, we applied the fixed assumed percentage growth to get the GDP per head when the second innovation occurred. If more than one innovation is imputed to occur in the same year, that is, if the growth rate in any year is greater than the fixed percentage defining an innovation, a log linear interpolation was employed.

2. Economic Theory

In the Aghion and Howitt [1992] framework, a stochastic sequence of innovations increases instantaneously and permanently on each arrival, the GDP per person in the economy. In its
simplest version, the size of each innovation is exogenous, fixed as a percentage of GDP. If the economy is in steady state, called a stationary stochastic equilibrium (SSE), the mean rate of arrival of innovations is a constant, generated by a constant number of workers put to research, and the stochastic process has no memory in the sense that the probability of an innovation depends only on the current research effort and not on the accumulated number of innovations.

The notion of an innovation here can be criticized in a fashion analogous of Joan Robinson’s objection to Solow’s creation of an aggregate physical capital stock. An innovation for us is the result of several technological or institutional changes and so cannot be linked to any single observable event in the real world, just as Solow’s growing $K$ is the result of several observed acts of investment in all sorts of stuff. In his theory, if there are no innovations, growth stops when capital accumulation is just sufficient to equip each additional worker with the same endowment as the existing ones. Intuitively, if the rate of capital accumulation is such that growth always stops before the arrival of the next innovation, then the increment to GDP per person between any two of these stopping points or temporary steady states would be due to an innovation, and the time between innovations will be determined by the stochastic process for innovations and not by the rate of capital accumulation. Here we are assuming that these temporary steady states precede or at most coincide with the arrival of each innovation.

It should be noted that in our interpretation, each economy’s $\lambda$ could be a composite of inheritance and individual and social choice at start-up. That is, it may have inherited low research productivity but it can compensate for this with more people doing research (see De Castro and Gonçalves [2001, 2003a]). Exactly what these folk do is not easily defined but in the metaphor of this paradigm, they would devote their energies to finding ways to increase labor productivity, either directly or indirectly.

3. Statistical Methods

There are several definitions of the simple Poisson process in the stochastic literature (Ross [1983] Chap. 2), depending on how the probability law governing the events is specified. Perhaps the one most suited to the statistical analysis here is derived from assumptions which imply that the inter-arrival times of the events, which are innovations in this application, are iid with a negative exponential distribution. However, we do not use this implication directly in our hypothesis testing. A renewal process is a natural generalization derived from this approach to the Poisson in that the negative exponential is replaced by an arbitrary but stationary distribution (Ross [1983] Chap. 3). If the mean arrival rate varies with time, $\lambda(t)$, the Poisson process is said to be non-homogeneous. If it varies with time and the accumulated number of innovations we say it is mixed (see Grandell [1997] for more details).

We assume that there existed at the start-up of growth, a theoretical (mixing) distribution for potential, initial, mean arrival rates of innovations, $V(\lambda)$, from which each economy drew its Poisson parameter, $\lambda$. If the economy then holds this parameter constant for all time up to $T$, it will exhibit a homogeneous Poisson process. If the theoretical distribution for the $\lambda$ is degenerate and its draw held constant, then each economy will exhibit a simple (homogenous) Poisson process, and the theoretical distribution for the innovation counts of the ensemble of economies at time $T$, $N(T)$, and therefore the world distribution of income across economies will be a Poisson distribution with mean $\lambda T$. 
Other stochastic methods have been used to study the real world distribution generated by the trajectories since 1950. In particular, Quah [1997] followed by others introduced Markov chain techniques into the analysis of these data. However, the economic theory behind the estimated transition matrices is not clear. Specifically, De Castro and Gonçalves [2001, 2003] showed that dominant diagonals in the transition matrices can result from each economy following a theoretical, stationary but quite general, stochastic process starting from a degenerate initial world income distribution (see also De Castro [1999, 1993]). Further, since the terminal Markov states (very poor, poor, middle etc), like the initial ones, are also defined using each economy’s position in the observed terminal distribution, one can have a situation in which the world may be converging in the absolute sense of reduced dispersion across economies, yet the transition matrix may still have dominant diagonals, meaning poor countries are categorized as remaining poor even though they are catching up rapidly with the rich.

3.1 How the Poisson process becomes a Random Walk

Since the statistical methods used here come from the actuarial sciences, we show now some connections to the more familiar techniques used in time-series econometrics. The basic difference is that the former treats with the occurrence of random events, here innovations that increase instantaneously the GDP per person, while the time-series methods deal with direct observations of these data but at regular intervals.

In the simplest model posed by Aghion and Howitt [1992], if the economy is in a stationary, stochastic equilibrium (SSE), then GDP per person, \( Y(t) \), follows:

\[
Y_{\tau+1} = \gamma Y_{\tau} \quad ; \quad \gamma > 1 \text{ constant, exogenous,} \quad \ldots (1)
\]

where \( \gamma \) represents the constant proportionate increase caused by each innovation; \( \tau \) is the innovation counter, \( \tau = 0, 1, 2 \ldots \).

In the SSE, innovations arrive in a (simple) Poisson process, with constant parameter, \( \lambda \) the mean rate of arrivals. Thus the arrival times, \( t_1, t_2, \ldots, t_{\tau} \), are such that the time intervals between arrivals, \( W_{\tau} = t_{\tau} - t_{\tau-1} \), have a negative exponential distribution, with parameter \( \lambda \).

But the GDP per person \( Y(t) \), is observed only at fixed times, \( t, t+1, t+2 \ldots \).

Let \( n(t) \) be the number of innovations between times \( t, t+1; n(t) \) would have a Poisson distribution, parameter \( \lambda \), constant if the economy is in a SSE.

\[ \Rightarrow E[n(t)] = \lambda = Var[n(t)] \]

Equation (1) can be transformed to:

\[
\ln Y(t+1) = \ln Y(t) + d(t) \quad ; \quad t = 0, 1, 2, \ldots \text{ time periods} \quad \ldots \ldots (2)
\]

Where \( d(t) = (\ln \gamma) n(t) \); \( E[d(t)] = (\ln \gamma) E[n(t)] \)

Define \( e(t) = d(t) - \lambda \ln \gamma \) \( \Rightarrow E[e(t)] = (\ln \gamma) E[n(t)] - \lambda \ln \gamma = 0 \)

\[ Var[e(t)] = (\ln \gamma)^2 Var[n(t)] = (\ln \gamma)^2 \lambda \]

Equation (2) becomes: \( \ln Y(t+1) = \ln Y(t) + \lambda \ln \gamma + e(t) \quad ; \quad t = 0, 1, 2, \ldots \text{ time periods} \quad \ldots \ldots (3) \)

Define \( y(t) = \ln Y(t) \) and \( \varepsilon(t) = e(t-1) \).

Equation (3) becomes:

\[ y(t) = y(t-1) + a_0 + \varepsilon(t); \quad a_0 \equiv \lambda \ln \gamma \]

which is a random walk with constant positive drift.

The solution is:

\[ y(t) = y(0) + a_0 t + \sum_{i=1}^{t} \varepsilon(i) \]

\[ \ldots \ldots (5) \]

Random walk models have a unit root. Each shock \( \varepsilon(i), i=1,2,\ldots,t \) has a permanent effect on \( y(t) \) as demonstrated in equation (5). The only difference here with standard random walk models
is that the shocks are not normally distributed but are Poisson variates, derived from the $n(t)$ distribution.

Shively [2001] tested the same US series, real GDP per person, 1869-1996, and rejected the unit root null. Thus he concluded that the series had shocks which are temporary deviations from trend. Our results below for the US came to the equivalent conclusion since we found that the US was on average never far from its long-term growth rate. It was trend stationary throughout the trajectory.

### 3.2 The tests for mixed Poisson processes

The procedure used follows Albrecht [1982] who provides theorems which prove that the goodness-of-fit tests given in the literature for homogeneous Poisson processes are in fact tests for mixed Poisson processes (e.g. Parzen [1962] pp.141-3; Cox and Lewis [1966] pp.153-8). He then argues that if an empirical point process passes any one of these tests, then the hypothesis being accepted is that it is a mixed Poisson process.

If a process passes one or more of these and in addition we want to test whether the mean arrival rate is constant, then a second-stage test must be applied to see if it is also a renewal process. This argument makes use of a theorem which states that the only mixed Poisson processes which are also renewal, are the homogeneous Poisson (for a proof see McFadden [1965]).

We applied two separate alternative goodness-of-fit tests to the data, both based on the theorem for homogeneous Poisson processes that, given $N(T)=n$, the $n$ occurrence times $T_1< T_2<...< T_n$ in $[0,T]$ are random variables having the same distribution as if they were order statistics corresponding to $n$ independent random variables, $U_1,...,U_n$ uniformly distributed on $[0,T]$ (see Parzen [1962] theorem 4A, p.140).

The two methods used in the time series of events data are:

(i) From the central limit theorem, for moderately large values of $n$, the sum

\[ S_n = \sum_i U_i \]

of $n$ independent random variables, each uniformly distributed on $[0,T]$ may be considered to be normally distributed with mean, \( E[S_n] = nE[U_1] = nT/2 \) and variance \( Var[S_n] = nVar[U_1] = nT^2/12 \); and from theorem 4A, if the process is Poisson, the sum of the times at which the $n$ events occurred is (approximately) normally distributed with the same mean and variance. Thus a test for Poisson is simply whether this observed sum is within the usual confidence interval for the normal distribution.

(ii) The second method applies an uniform conditional test using the Kolmogorov-Smirnov (K-S) statistic to examine if the unordered occurrence times $T_i/T$ are a sample from the uniform distribution over $[0, 1]$. The definition of the statistic is

\[ D_n \sqrt{n} \]

where:

\[ D_n = \max_{1 \leq i \leq n} \left| \frac{i}{n} - \frac{T_i}{T} \right| \]
and where \( i/n \) is the distribution function specified in the null hypothesis, a uniform distribution here, and \( T_i/T \) is the observed distribution.

If the data were to pass these tests, and we want to decide if the Poisson process has a constant, \( \lambda \), then one possible way to check for a renewal process is the serial correlation test (Cox and Lewis [1966] p.164) since, under the null hypothesis that it is renewal, the correlation coefficients \( \rho(W_i, W_{i+1}) \) of the successive time intervals between arrivals are zero and are independent of the time period \( i \), under stationary alternatives including the mixed Poisson process.

Albrecht [1982] also argues that no non-homogeneous Poisson process could have passed the first-stage tests for mixed Poisson because:

(i) A non-homogeneous Poisson process with arrival rate \( \lambda(t) \) has the property, given \( N(T)=n \), that the occurrence times \( T_1, T_2, \ldots, T_n \) have the same distribution as if they were the order statistics corresponding to \( n \) independent draws, each having density:

\[
\frac{\lambda(t)}{\int_0^T \lambda(s)ds}
\]

(See Parzen [1962] exercise 4B p.143. Proof not given)

(ii) And since a point process which is both a mixed Poisson and a non-homogeneous Poisson must satisfy:

\[
\frac{\lambda(t)}{\int_0^T \lambda(s)ds} = \frac{1}{T}
\]

we must have that \( \lambda(t) \) is a constant and so the process must be homogeneous.

To sum up in terms which are coherent with growth theory, an economy whose GDP per person since start-up exhibits a process which is both mixed Poisson and a renewal, can be said to be history dependent in the sense that the mean arrival rate of innovations with which its growth commenced has remained constant ever since. One may want to say that in such an economy, most of the observed structural changes like industrialization, urbanization and education and population levels are probably consequences of a more fundamental, stationary, stochastic growth process. As in Bils & Klenow [2000], it is growth that would cause high education levels and not the other way around. The true cause of growth remains moot.

3.3 The estimation of the world \( \lambda_i(0) \) mixing distribution in 1800

In this procedure, the only data needed are the innovation count for each economy \( i \) \((i=1,2,\ldots,m; m=134)\) in year 2000, \( N_i(T) \), \( T=200 \). The Summers-Heston GDP per person in 2000, together with the De Castro and Gonçalves [2001, 2003] conversion method provide these counts.
In the actuarial science literature up to about 1980, there seems to have been several methods for estimating the mixing distribution, $V(\lambda)$, of a mixed Poisson process, based on assumed, special parametric types derived from the Polya, Hoffman and double-Poisson processes. Here, we follow Albrecht’s [1982a] proposal to estimate a general, but discrete, mixing distribution. The only assumptions are that it is a step function with a finite number of steps (classes), and that this number, $s$, is given. Thus the $2s-1$ parameters to be estimated are the boundaries of the classes, $0<\lambda_1<...<\lambda_s$ and their respective relative frequencies $b_1, ..., b_s$ with $b_j>0, \forall j, \sum_j b_j = 1$.

Although Albrecht [1982a] gave two methods for estimating the mixing distribution from the $N_i(T)$ event counts, we implemented only one so far in the research. For reasons of space we will give just a bare outline of the numerical procedure which yields only a first estimate which can be refined later into a BAN-estimator (not done here).

A mixed Poisson process $N(T)$ with mixing distribution $V(\lambda)$ of the random variable $\Lambda$ can be shown to satisfy:

$$E[N(T)|N(T)-1]...E[N(T)-k+1] = T^k E[\Lambda^k]$$

(*)

Now $E[\Lambda^k] = \sum_{j=1}^{s} b_j \lambda_j^k$, by our hypothetical $V(\lambda)$ distribution.

Put $F_k = [N(T)|N(T)-1]...E[N(T)-k+1)]/T^k$

Then equation (*) can be written:

$$E[F_k] = E[\Lambda^k] = \sum_{j=1}^{s} b_j \lambda_j^k$$

(**)

We can use the observed sample count $N_i(T), i=1,..,m$ economies to obtain an estimate of $E[F_k], \tilde{F}_k$, for the first $(2s-1)$ sample counterparts thus:

$$\tilde{F}_k = \left[ \sum_{i=1}^{m} N_i(T)|N_i(T)-1]...E[N_i(T)-k+1)] \right]/mT^k, k=1,...,2s-1$$

and substitute these in the LHS of equation (**) to get:

$$\sum_{j=1}^{s} b_j \lambda_j^k = \tilde{F}_k, k=1,...,2s-1$$

(***)

Thus we now have $(2s-1)$ equations for the $(2s-1)$ unknowns $b_j, \lambda_j$ which are linear in $b_j$.

Albrecht [1982a] uses this linearity to show that the $\lambda_j$ can be obtained as the ordered roots of the $s$th degree polynomial:

$$x^s + B_1 x^{s-1} + ... + B_s x + B_0 = 0$$

the coefficients of which are obtained from the unique solution to the system of linear equations:

$$\sum_{k=0}^{s-1} \tilde{F}_{k+j} B_k = -\tilde{F}_{s+j}, j=0,...,s-1, where \tilde{F}_0 \equiv 1$$

The $\lambda_j$ so obtained can then be substituted into the first $(s-1)$ equations (***) above to solve a linear system for the $b_j, j=1,..,s-1$. 
4. The data sources for Brazil’s GDP per person, 1822-2000

Although there have been several published estimates of growth rates for Brazil since start-up, the official statistical agency (IBGE) has been reluctant to place any confidence in them. The claim is that the primary sources are very sparse. However, since Haddad’s [1974] pioneering study for 1900-47, it is commonly agreed that the economy grew faster than the US in the 20th century until about 1975. For this to be compatible with the still relatively low income per head in 1950 when the Summers-Heston data start (about 16% of the US), the economy had to begin the century exceedingly poor. In 1900, Brazil’s income per head was on par with India’s and China’s - $436, $378 and $401 respectively, in 1985 dollars, compared to $3,101 for the US in 1890 (see Barro & Sala-i-Martin [1995] Tables 10.2, 10.3).

Celso Furtado’s classic ([1963] chap.25) gives an estimate for 1900-1950 that is not consistent with these sources. Further, Furtado claims (p.163) that for both 1850-1900 and 1900-1950, Brazil’s income per head grew steadily at 1.5% per year, a rate that is faster than “the average for the economies of Western Europe”. Since, as we have seen, Brazil was very poor in 1900, a 1.5% growth rate for 1850-1900 projected backwards from 1900 would soon put it below subsistence in the mid-19th century, an implausible situation. Our table below also has implausibly high growth rates, for 1822-2000 (1.55%) and 1850-2000 (1.76%), when compared to the US with 1.68% (1820-1992) and 1.67% (1800-1989) per year calculated respectively from Maddison [1995] (table 1.3) and Engerman and Sokoloff [1997] (table 10.5 p.270). The error, we will suggest, is mainly due to under-estimates of the income levels for the period 1822-1850/60.

Table 1: Growth rates, 1800-2000

<table>
<thead>
<tr>
<th>Period</th>
<th>Brazil GDP pp</th>
<th>USA GNP pp</th>
</tr>
</thead>
<tbody>
<tr>
<td>1822-2000</td>
<td>1.55%</td>
<td>1800-1989 1.67%</td>
</tr>
<tr>
<td>1822-1950</td>
<td>1.01%</td>
<td>1820-1950 1.56%</td>
</tr>
<tr>
<td>1822-1900</td>
<td>0.16%</td>
<td>1800-1913 1.60%</td>
</tr>
<tr>
<td>1822-1850</td>
<td>0.44%</td>
<td>1800-1850 1.10%</td>
</tr>
<tr>
<td>1850-2000</td>
<td>1.76%</td>
<td>1869-1996 1.74%</td>
</tr>
<tr>
<td>1850-1950</td>
<td>1.18%</td>
<td>1869-1950 1.66%</td>
</tr>
<tr>
<td>1850-1900</td>
<td>0.01%</td>
<td>1869-1900 1.73%</td>
</tr>
<tr>
<td>1900-2000</td>
<td>2.64%</td>
<td>1900-1996 1.76%</td>
</tr>
<tr>
<td>1900-1950</td>
<td>2.34%</td>
<td>1900-1950 1.89%</td>
</tr>
<tr>
<td>1950-2000</td>
<td>2.93%</td>
<td>1950-1996 1.89%</td>
</tr>
<tr>
<td>1950-1975</td>
<td>4.48%</td>
<td>1950-1975 1.52%</td>
</tr>
<tr>
<td>1975-2000</td>
<td>1.39%</td>
<td>1975-1996 2.10%</td>
</tr>
</tbody>
</table>

Sources: Brazil: Table 1 in Appendix and Summers-Heston world tables; USA: 1800- 1989 Engerman & Sokoloff [1997]; 1869-1996 Shively [2001]
The sources of our complete series were as follows. For 1950-2000, they are the Summers-Heston data, which are given in 1996 US dollars corrected for purchasing power parity (PPP). For 1850-1949, we used the annual indices for real per capita GDP measured in Brazilian currency and calculated by Goldsmith [1986], to get growth rates and worked backwards from the 1950 GDP value given by Summers-Heston. This is a very rough method of trying to maintain some semblance of a PPP correction but it is superior, we think, to the method of Leff [1982], Contador e Haddad [1975] and others of using the real exchange rate to take the data to US dollars. For example, Contador e Haddad [1975] has the US, in both 1860 and 1970, with 10 times the income per head of Brazil.

Finally, for 1822-1849, De Castro and Gonçalves [2003a] applied Leff’s [1972] method for calculating the growth rates of real GDP per person from his series for annual, deflated currency stock and his population data (see Leff [1982] p.241) to obtain a positive average annual growth rate for GDP per person of 0.44%, 1822-1850. These annual growth rates were then used to project backwards from the 1850 GDP per person level obtained using the Goldsmith indices.

Despite the under-estimates in our 1822-1850/60 income levels, the plausible adjustments we suggest below for this period should not alter the main results of our statistical tests (see section 5). They ought not because it was generated, we think, by the long period of stagnation in the mid to late 19th century. In fact, the period 1850/60 to 1900/1913 is shown with absolute declines by two of our primary sources, Contador e Haddad and Goldsmith so these declines deserve to be treated now as definitive. Leff, our third primary source, did not separate out his 19th century national growth rate into its two halves.

Both Maddison and Engerman and Sokoloff give estimates for Brazil’s long-term growth rate at around 1.0%, respectively 1.09% (1820-1992) and 0.93% (1800-1989), which we find are about right but for slightly different reasons. Both under estimate Brazil’s GDP per person in their terminal years, when compared to the Summers-Heston data because, we presume, of inadequate PPP corrections. Maddison also under-estimates his initial value (put at about half the US) which, if not corrected at the same time, would yield a much higher growth rate for 1820-1992 of 1.33%.

Engerman and Sokoloff [1997] is the only published source consistent with roughly equal incomes at start-up, $738 and $807 for Brazil and the US respectively in 1800, 1985 dollars (table 10.5 p.270). Their table shows Brazil growing to $901 in 1850 and falling to $700 in 1913, less than for 1800. This fall is consistent with the experience of most other Atlantic slave economies for which we have direct estimates of GDP data e.g. the US south, Jamaica and British Guiana (see De Castro [2004]).

It seems then that our method under-estimates significantly Brazil’s GDP per person around 1822, and somewhat less so around 1850/60. The main reason for this, we surmise, is that all our primary sources for the period up to 1900 used either international trade and/or monetary data. For example, Leff used deflated money stocks to derive what he called “monetized per capita income”. If, as he asserted, “output was growing at a higher rate in the monetized (sector)” ([1982] footnote 21, p361), this further reinforces our position since the omission of the slower growing, non-monetized incomes of mainly subsistence farmers would lead, with our method, to lower estimates of initial GDP. Though slaves received incomes mostly in kind, these may have been counted in the monetary and trade data from the revenue side of the plantations.

As a first pass at suggestions for revision, let’s put the GDP per person in 1996 dollars at $700 to $800 in 1822, yielding a growth rate to 2000 of 1.24 to 1.32% per year. In 1850/60, this should rise, with our and Leff’s rates, to between $800 and $900. Now let it fall to the $512 we
give for 1900, which low figure is consistent with the fairly consensual, high 20th century rates. The resulting negative growth in the late 19th century would be about 1.0% per year, slightly higher than the minus 0.80% for 1870-89 calculated from Goldsmith (pp 20-21), and much higher than the minus 0.40% for 1861-1900 he derived from data in Contador e Haddad.

However, our suggested correction would still leave the 1822 GDP per person somewhat lower than the US, set at $1,242, an estimate based on the Summers-Heston 1950 figure of $10,350, projected backwards at 1.67%. This gap is closer to Maddison’s and thus contradicts both Engerman & Sokoloff and our general claim of roughly equal incomes at start-up. If we were to reduce this initial gap by putting Brazil even higher, at say $1,000, and to maintain the 1822-50 growth rate at our and Leff’s 0.44%, this would reduce the 1822-2000 rate to the more plausible 1.11%. But the fall between 1850 and 1900 would be even more drastic, at 1.57% per year.

5. The results

5.1 Brazil, 1822-2000; 1889-2000; USA, 1869-1996

For Brazil’s GDP per person, 1822-2000, the main result is that its growth is not a mixed Poisson process. Both goodness-of-fit tests were applied, the K-S statistic (see table below), and the test based on the central limit theorem. For the truncated series 1889-2000, chosen for reasons given below, the growth trajectory passed the latter test easily and the K-S at the 1% level, but just failed it at the 5% (see table 2 below). For a comparison we tested the US GNP per person, 1869-1996, obtained from Shively [2001], and this also is a mixed Poisson process, having passed both tests.

For the test based on the central limit theorem, the period 1822-2000 yielded a sum of the times at which the n events occurred way outside the confidence limits for the normal distribution, with a standard normal z value of 4.85. For 1889-2000, the z value was minus 0.0744 well within the limits, as it was for the US data, with z value 0.5313.

The asymptotic significance values for the K-S statistic when n ≥ 100 are given in Cox and Lewis [1966, p.258] as: upper 5% point: 1.358; upper 1% point: 1.628. If K-S is less than these values, the null hypothesis is to be accepted, i.e. the process is a mixed Poisson. The results for this test are given in table 2.

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<tr>
<th>Country, Period</th>
<th>K-S</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil, 1822 - 2000</td>
<td>3.68</td>
<td>Reject null</td>
</tr>
<tr>
<td>Brazil, 1889 - 2000</td>
<td>1.41</td>
<td>Rjct/acpt</td>
</tr>
<tr>
<td>USA, 1869 - 1996</td>
<td>0.001</td>
<td>Accept null</td>
</tr>
</tbody>
</table>

Table 2: Test for mixed Poisson process, Kolmogorov-Smirnov statistic

The K-S statistic depends on the largest of the deviations from an assumed uniform distribution for the successive time intervals between innovations and provides a clue to the interpretation of the results. The main cause of the rejection for Brazil, 1822-2000, is its long period of stagnation in the second half of the 19th century, relative to its high growth for most of the 20th. For example, our conversion using the 3% definition for innovation size gave an imputed interval of 86.5 years between the 14th and 15th innovation (see table 2 in the appendix).
In growth language, the economy took 86.5 years to increase GDP per person by 3%. This was the main reason we split the series in the late 19th century. The year Brazil became a republic, 1889, was chosen partly because this change of regime is closely linked to the completion of the abolition of slavery in 1888, a protracted process started around 1850.

We claim therefore that an economy will pass the K-S test if it exhibited sustained, though not necessarily high growth throughout the period studied, since this is the type of trajectory which would generate less variation in the inter-arrival times because of the way we derived these from the GDP data. So an India or a China, 1800-1980 say, may well pass the tests, as may the US (we know it passed for 1869-1996), because these are based implicitly on the variation in the annual growth rates, not their mean over the whole trajectory. Thus whereas it is true that stagnation in the late 19th century is almost definitive of the economies which became the future third world, the more precise tentative conclusion of this study would seem to be that economies which have had trajectories since start-up with both stagnation and growth spurts are less likely to pass the tests for mixed Poisson processes.

5.2 If both mixed Poisson and Renewal, then there is history dependence

For the trajectories that have passed the tests for mixed Poisson, we applied the second-stage serial correlation test outlined in Section 3.2 to see if they are also renewal processes, to confirm the hypothesis of history dependence. Both Brazil, 1889-2000, and the US, 1869-1996, showed no significant serial correlation in the lengths of the successive intervals between innovations and the hypothesis can be confirmed. Despite this conclusion, at the end of the 20th century these two economies have very different levels of GDP per person, illustrating our point that history dependent growth does not imply equal terminal income levels, even if our earlier assertion is accepted that initial levels for all economies were more or less equal.

5.3 Test for a renewal process in Brazil's GDP per person, 1822-2000

Brazil’s complete growth trajectory did not pass the test for a mixed Poisson process. We decided to go ahead anyway and apply to it the serial correlation test for a renewal process. We found no significant serial correlation in these data. We can conclude therefore that Brazil’s growth is a renewal process, but since it is not also a mixed Poisson, we cannot say it is history dependent in our sense. Put another way, the distribution of the intervals for the completion of fixed percentage increments to GDP per person is stationary but unknown. This result in effect would imply that the arrival rate of innovations has varied over the period 1822-2000 in some unspecified manner still to be identified.

So if history dependent growth is taken to be a homogeneous Poisson process, of which its constant, mean arrival rate of innovations was fixed at start-up, then Brazil’s growth trajectory since political independence does not exhibit history dependence.

5.4 The world’s arrival rate ($\lambda_i(0)$) mixing distribution in 1800

The procedure requires that the number of classes to be estimated be fixed exogenously. Here $\lambda_j$, $b_j$ are respectively the boundaries and relative frequencies of each class in the estimated distribution. However, for none of the definitions of size of innovations, from 1% to 7% growth of GDP per person, were we able to obtain a valid estimated distribution for more than 4 classes (see tables below), as some $\lambda_j$ were negative or complex numbers. Remember they are the roots of a polynomial (section 3.3 above). With exactly 4 classes, only sizes 1% and 2% yielded valid distributions.
Table 3.a: The estimated mixing distribution in 1800, 2 classes

<table>
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<tr>
<th>Size of innovation</th>
<th>1%</th>
<th>2%</th>
<th>3%</th>
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<th>6%</th>
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<td>$\lambda$</td>
<td>$b$</td>
<td>$\lambda$</td>
<td>$b$</td>
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<td>0.54</td>
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<td>0.28</td>
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Table 3.b: The estimated mixing distribution in 1800, 3 classes

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<th>3%</th>
<th>4%</th>
<th>5%</th>
<th>6%</th>
<th>7%</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$b$</td>
<td>$\lambda$</td>
<td>$b$</td>
<td>$\lambda$</td>
<td>$B$</td>
<td>$\lambda$</td>
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Table 3.c: The estimated mixing distribution in 1800, 4 classes

<table>
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As the definition of innovation size was increased, the data conversion to innovation count would yield fewer recorded innovations in the Summers-Heston distribution for 2000. This would mean that the estimated arrival rates should decrease with innovation size. This is what we see in the decreasing range for the $\lambda_j$ in the estimated mixing distributions in table 3.

However, as the values of these $\lambda_j$ boundaries for the classes decreased, the distributions became more negatively skewed, meaning that the higher values of $\lambda_j$ became more likely. Put another way, the smaller the number of innovations imputed to have occurred since 1800, the greater the chance that a higher value for $\lambda_i$ would have been drawn by each economy $i$. Thus, if the assumption of homogeneous Poisson used for the estimation is true, and if the view of narrative economic history shall be that a few, isolated, large innovations are the main engine of growth, then it is likely that only a small group will have drawn a low value for $\lambda_i$, and the main cause of the dispersion 200 years later would be sheer luck, not of the draw, but of outcomes from nearly equal mean arrival rates of innovations, in the same stationary process shared by almost all economies.
6. References


Furtado, Celso (1963), *The economic growth of Brazil*, Univ. of California Press (original 1959).


6 tables and 3 graphs follow in the appendix
Appendix

Table 1 GDP per person (1996 US$): Brazil 1822 - 2000

<table>
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<tr>
<th>Year</th>
<th>GDP</th>
<th>Year</th>
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<th>Year</th>
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1850 – 1950: Goldsmith [1986]  
1822 – 1850: calculated, based on data and method in Leff [1972]
Table 2: Arrival times of innovations: Brazil 1822 – 2000; $T_0 (1822) = 0$; 3% size

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Graph 1 – Observed vs. Uniform Distribution of arrivals: Brazil, 1822 – 2000; 3% size
Table 3: Arrival times of innovations: Brazil 1889 – 2000; $T_0 (1889) = 0$; 3% size

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Graph 2 – Observed vs. Uniform Distribution of arrivals: Brazil, 1889-2000; 3% size
Table 4: Logarithm of real GNP per person: USA, 1869-1996

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Source: Shively [2001]

Table 5: Arrival times of innovations: USA, 1869-1996; $T_0 (1869) = 0$; 3% size

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Graph 3 – Observed vs. Uniform Distribution of arrivals: USA, 1869-1996; 3% size
Table 6: GDP per person, 134 countries, in 2000 (1996 US$)

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Source: Summers-Heston world tables

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<td>06-12-2013</td>
<td>On the Representation of Incomplete Preferences under Uncertainty with Indecisiveness in Tastes, Gil Riella</td>
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