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An Introduction to Evolutionary Game Theory

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1. Evolutionary Game theory

Evolutionary Game theory: John Maynard Smith and George Price.

Mathematical biology: Lotka and Volterra model: dynamics of populational growth.

Game Theory: John Nash, Von Neumann and Mongestern: strategic thinking.

	S	T
S	a,a	b,c
T	c,b	d,d

N: number of individuals

N_S: number of individuals playing S

N_T: number of individuals playing T

$N = N_S + N_T$

Expected utility of playing S:

$$U_S = a.x + b.(1-x)$$

where $x = \dfrac{N_S}{N}$

Expected utility of playing T:

$$U_T = c.x + d.(1-x)$$

Average utility:

$$\overline{U} = [a.x + b.(1-x)]x + [c.x + d.(1-x)](1-x)$$

Dynamic Replicator

$$\frac{\dot{x}}{x} = U_S - \overline{U}$$

$$\frac{\dot{y}}{y} = U_T - \overline{U}$$

$$\frac{\dot{x}}{x} = a.x + b.(1-x) - \left[a.x + b.(1-x)\right]x - \left[c.x + d.(1-x)\right](1-x)$$

$$\frac{\dot{x}}{x} = (1-x)\left\{\left[a.x + b.(1-x)\right] - \left[c.x + d.(1-x)\right]\right\}$$

$$\dot{x} = x(1-x)\left\{\left[a.x + b.(1-x)\right] - \left[c.x + d.(1-x)\right]\right\}$$

$$\dot{x} = xy\left\{a.x + b.y - c.x - d.y\right\}$$

$$\dot{x} = xy\left\{(a-c)x + (b-d)y\right\}$$

By symmetry, we conclude that:

$$\dot{y} = xy\left\{(c-a)x + (d-b)y\right\}$$

Example: Hawk and dove:

Given that the resource is given the value V, the damage from losing a fight is given cost C:

- If a Hawk meets a Dove he gets the full resource V to himself
- If a Hawk meets a Hawk – half the time he wins, half the time he loses…so his average outcome is then V/2 minus C/2
- If a Dove meets a Hawk he will back off and get nothing - 0
- If a Dove meets a Dove both share the resource and get V/2

	H	D
H	$(V-C)/2, (V-C)/2$	$V,0$
D	$0,V$	$V/2,V/2$

Hawk and dove with $V > C$:

Then (H,H) is a NE. The dynamic replicator in this case is given by: $\dot{x} = \frac{1}{2}x(1-x)\{Cx - V\}$. The interior solution does not hold since $x^* = \frac{V}{C} > 1$. Note that $\dot{x} < 0$ in the interval (0,1). Then $x = 0$ is an ESS while x=1 is not.

V =4 and C= 2.

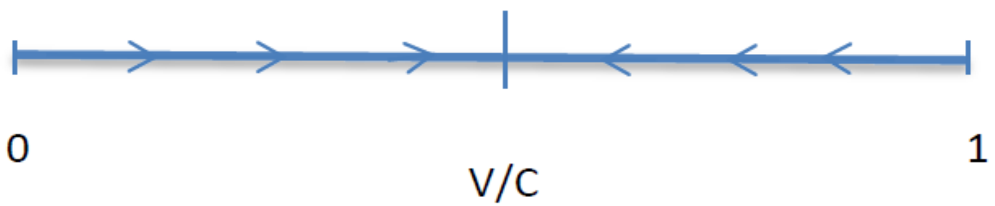
	H	D
H	1, 1	4,0
D	0,4	2,2

Hawk and dove with $V < C$:

V = 4 and C = 6

	H	D
H	-2, -2	4,0
D	0,4	2,2

NE in pure strategies: (H,H) and (D,D). The dynamic replicator is given by: $\dot{x} = \frac{1}{2}x(1-x)\{Cx - V\}$. But now the interior solution is admissible since $x^* = \frac{V}{C} < 1$. Note that $\dot{x} > 0$ in the interval (0,V/C) and $\dot{x} < 0$ in the interval (V/C,1). Then $x = 0$ and x=1 are not EES. The only EES is $x^* = \frac{V}{C} < 1$ and $1 - x^* = 1 - \frac{V}{C}$.



Acemoglu(2009, p. 114): Luck and Multiple Equilibria

		Everybody else	
		High investment	Low investment
Individual	High investment	y^H, y^H	$y^L - \varepsilon, y^L$
	Low investment	$y^L, y^L - \varepsilon'$	y^L, y^L

Expected utility of playing H:

$$U_H = y^H.x + (y^L - \varepsilon).(1-x)$$

where $x = \dfrac{N_H}{N}$

Expected utility of playing L:

$$U_L = y^L.x + y^L.(1-x) = y^L$$

Average utility:

$$\overline{U} = \left[y^H.x + (y^L - \varepsilon).(1-x)\right]x + \left[y^L\right](1-x)$$

$$\frac{\dot{x}}{x} = U_H - \overline{U} = y^H.x + (y^L - \varepsilon).(1-x) - \left[y^H.x + (y^L - \varepsilon).(1-x)\right]x + \left[y^L\right](1-x)$$

$$\dot{x} = x(1-x)\left\{\left[y^H - (y^L - \varepsilon)\right].x - 2y^L - \varepsilon\right\}$$

$$x^* = 0 \ \text{ ou } \ x^* = 1 \ \text{ ou } \ x^* = \frac{2y^L + \varepsilon}{y^L - y^H + \varepsilon}$$

We exclude $x^* = \dfrac{2y^L + \varepsilon}{y^L - y^H + \varepsilon}$

Asymmetric Games

Game of buyers and sellers:

	H	D
I	3,2	2,1
T	4,3	1,4

In this game, each seller can be either honest (H) or dishonest (D). Each buyer can either inspect (I) or trust (T). There is no Nash Equilibrium in pure strategies.

Let p the fraction of buyers who 'inspect' and q be the fraction of sellers who are 'honest'.

Expected pay-off of a seller playing H: $U_S^H = 2.p + 3.(1-p) = -p + 3$

Expected pay-off of a seller playing D: $U_S^D = 1.p + 4.(1-p) = -3p + 4$

Expected pay-off of a buyer playing I: $U_B^I = 3.q + 2.(1-q) = q + 2$

Expected pay-off of a buyer playing T: $U_B^T = 4.q + 1.(1-q) = 3q + 1$

Average pay-off of a seller

$$\overline{U}_S = q[2.p + 3.(1-p)] + (1-q)[-3p + 4] = q(2p-1) - 3p + 4$$

Average pay-off of a buyer

$$\overline{U}_B = p[q+2] + (1-p)[3q+1] = 3q + 1 - p(2q-1)$$

The dynamics replicator for p and q in this case is given by:

$$\frac{\dot{q}}{q} = U_S^H - \overline{U}_S = (1-p)(2p-1)$$

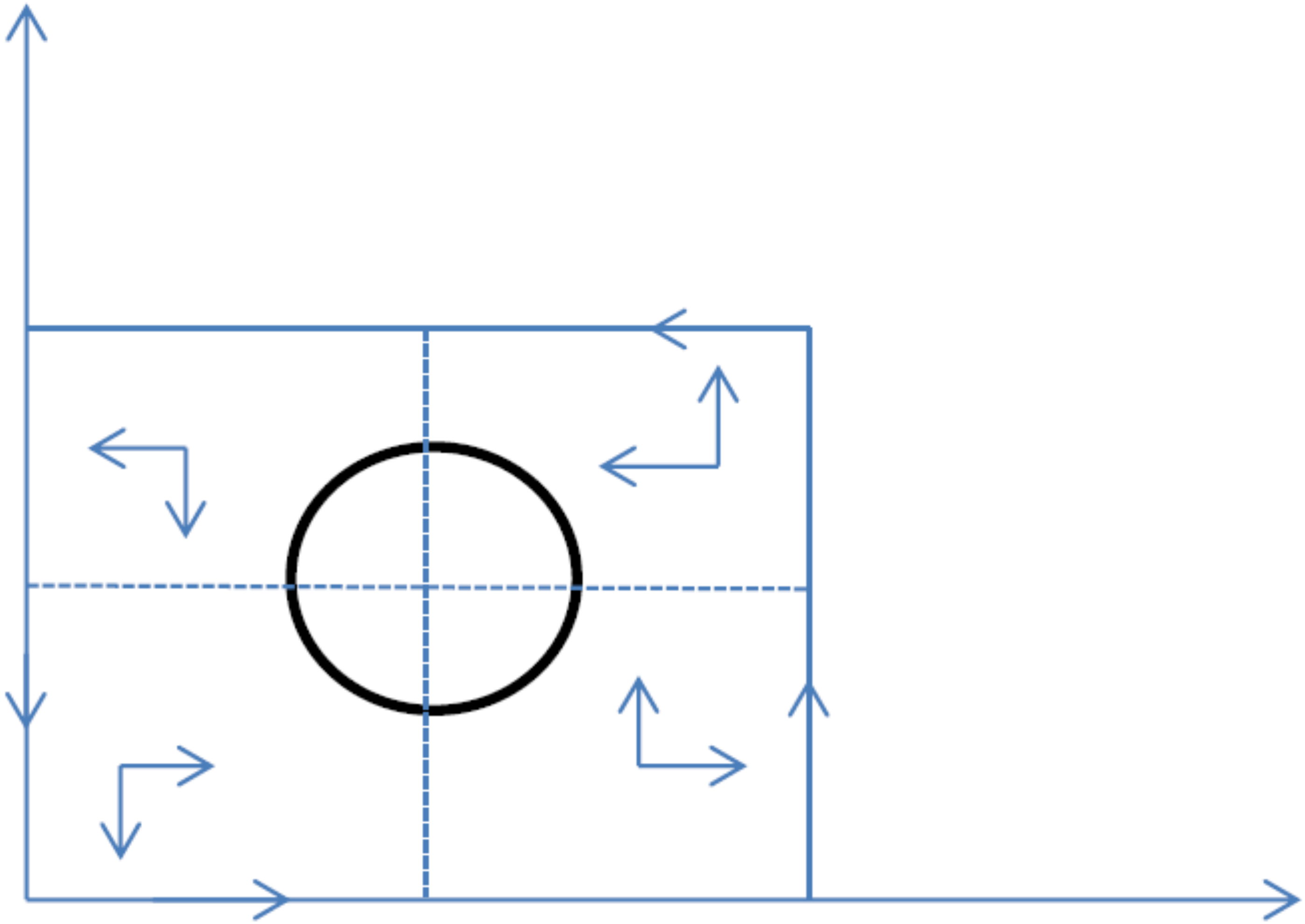
$$\frac{\dot{p}}{p} = U_S^H - \overline{U}_S = (1-p)(1-2q)$$

Then we have the following system of differential equations:

$$\begin{cases} \dot{q} = q(1-p)(2p-1) \\ \dot{p} = (1-p)(1-2q) \end{cases}$$

Evaluating the system in steady state yields the following solutions: (0,0), (0,1), (1,0), (1,1), (1/2,1/2). The Jacobian of the system is given by:

$$J(p,q) = \begin{bmatrix} (1-2q)(1-2p) & -2p(1-p) \\ 2q(1-q) & (2p-1)(1-2q) \end{bmatrix}$$



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An Evolutionary Game Theory Approach to the Dynamics of Labour Market: A Formal and Informal Perspective

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1. Introduction

Informality:

About 50% of the Brazilian Labor force is in the informal sector;

In Latin America this percentage varies from 30% to 70%;

- Excessive regulatory system that makes formal economy little attractive;
- Highly segmented in informality, yielding a negative impact on productivity levels.
- Fiscal evasion;
- Few protection by police and courts in cases of crime against property.
- No access to capital markets;
- Little access to other public services whether it be social security, training programs or facilitated credit.

1. Introdução (cont.)

Aims:

• To approach the labour market segmentation in a temporal evolution, starting from a microeconomic point of view of agents' choice making and going through a macroeconomic assessment of formal and informal sectors' dynamics;

• To delineate the optimal policy of taxation;

2. Literature Review

- Loayza (1996): end up jamming public infrastructure, bringing down productivity and affecting negatively the countries' growth.
- Almeida and Carneiro (2005): an increase in informality is the outcome of firms' search for greater flexibility in the economy.
- Hirschman (1970), who considers that workers and firms make implicit cost-benefit analysis when deciding to act in formal or informal sectors.
- Maloney (2004): workers choose informality due to a degree of dignity and autonomy that this type of work might offer.
- Perry et al. (2007): avoid complex schemes of tax payments, rigid labor legislation and difficult access to credit to invest on human and physical capital.

2. Literature Review

Matching and search models

- Pissarides (2000): rational firms and workers maximize their pay-offs, given the stochastic process that breaks up jobs and leads to the formation of new ones;
- Fagiolo et al (2004): present labour market dynamics in an agent-based evolutionary model;
- Richiardi (2006) formulates a non-equilibrium agent-based model, which allows a more comprehensive investigation of the labour market;
- Silveira and Sanson (2003): evaluate immigration as job allocation, following the Harris-Todaro theory.

3. The Model

Supply Side

Instantaneous expected utility, U_f, is assumed to be given by:

$$U_f = \sigma(1-\tau)w_f \qquad (1)$$

w_f: is the wage in the formal sector

τ: $0 < \tau < 1$, is the income tax.

σ: probability of finding a job in this sector.

In the informal sector his expected utility, U_i, is given by:

$$U_i = (1-\sigma)[(1-\rho)w_i + \rho(w_i - m)] \qquad (2)$$

w_i: is the wage paid in the informal sector,

ρ: $0 \le \rho \le 1$, is the probability that the worker faces of paying a fine, denoted by m,

Demand Side

Mortesen and Pissarides (1994): each firm hires only one worker, that is $l_f = 1$, and produces a fixed amount of product at a time, namely y_f. The price of the product is normalized to 1. Let it θ be the probability of a firm that chooses the formal sector to find a worker that decides to supply labour in this sector. Profit of the firm if it decides to operate in the formal sector is given by:

$$\Pi_f^e = \theta\left[y_f - (1+\gamma)w_f\right] \quad (3)$$

Where γ, $0 \leq \gamma \leq 1$: stands for the costs for being in the formal sector.

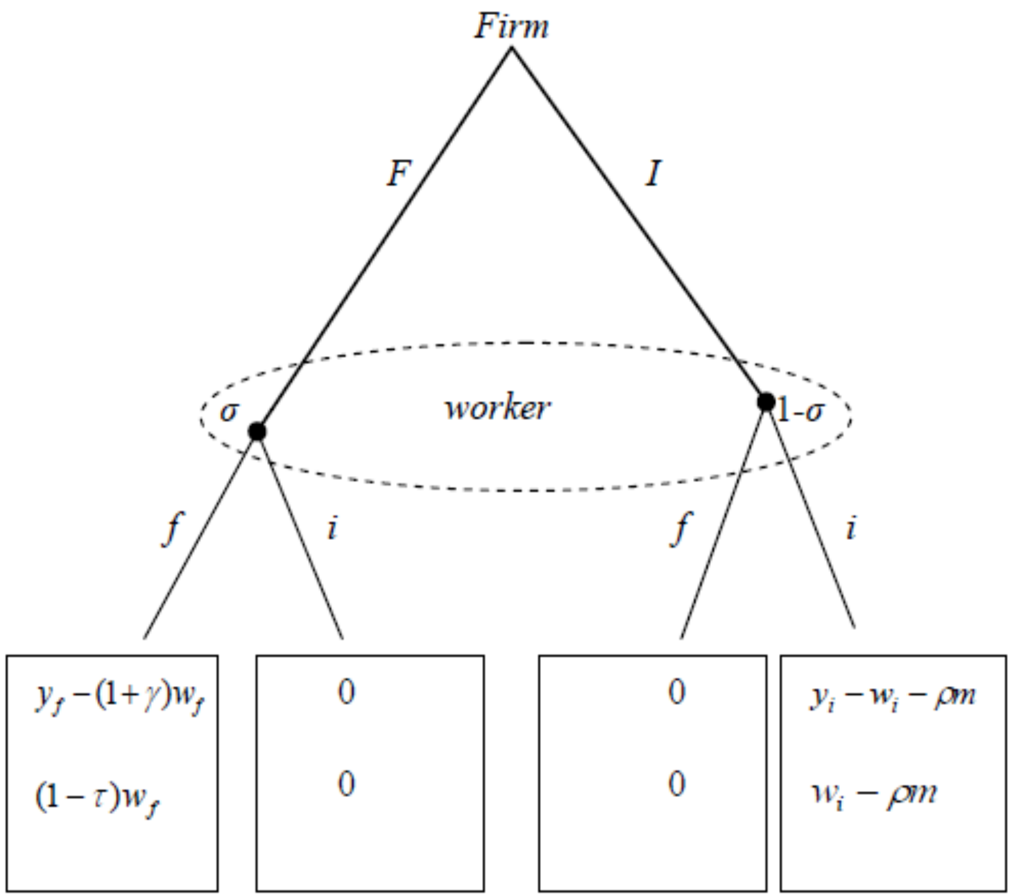
Informal sector: The firm is also assumed to hire only one worker, that is $l_i = 1$, but produces: $y_i < y_f$.

Profit of the firm in the informal sector is given by:

$$\Pi_i^e = (1-\theta)\{(1-\rho)[y_i - w_i] + \rho[y_i - w_i - m]\} \quad (4)$$

ρ, $0 \leq \rho \leq 1$, is the probability that the firm faces of paying a fine, expressed by m.

$$\Pi_i^e = (1-\theta)(y_i - w_i - \rho m) \quad (5)$$



Two pure Nash equilibria namely $\{F,f\}$ and $\{I,i\}$

A Mixed strategy equilibrium: both workers and firms randomly choose between being formal or informal.

3.2. Dynamic Replicators for the supply side:

$$\dot{N}_f = N_f\left[U_f - \overline{U}_{f,i}\right] \quad (6)$$

$$\dot{N}_i = N_i\left[U_i - \overline{U}_{f,i}\right] \quad (7)$$

$\overline{U}_{f,i}$: the average pay-off.

$$\dot{n}_f = n_f n_i\left\{\sigma(1-\tau)w_f - (1-\sigma)\left[w_i - \rho m\right]\right\} \quad (8)$$

$$\dot{n}_i = n_f n_i\left\{(1-\sigma)\left[w_i - \rho m\right] - \sigma(1-\tau)w_f\right\} \quad (9)$$

If (i) $\tau < \dfrac{\sigma w_f - (1-\sigma)\left[w_i - \rho m\right]}{\sigma w_f}$ or $\sigma > \dfrac{w_i - \rho m}{(1-\tau)w_f + w_i - \rho m}$ then $n_f = 1$, where all the labour force chooses the formal sector. If (ii) $\tau > \dfrac{\sigma w_f - (1-\sigma)\left[w_i - \rho m\right]}{\sigma w_f}$ or $\sigma < \dfrac{w_i - \rho m}{(1-\tau)w_f + w_i - \rho m}$ then $n_i = 1$, where all the labour force chooses the informal sector. If (iii) $\tau = \dfrac{\sigma w_f - (1-\sigma)\left[w_i - \rho m\right]}{\sigma w_f}$ or $\sigma = \dfrac{w_i - \rho m}{(1-\tau)w_f + w_i - \rho m}$, then $0 < n_f < 1$ and $0 < n_i < 1$, where mixed strategies prevail.

$\sigma = \eta_f$. In this case, expressions (8) may be rewritten as: $\dot{n}_f = n_f n_i\left\{\eta_f(1-\tau)w_f - (1-\eta_f)\left[w_i - \rho m\right]\right\}$ (8)’

Dynamic replicators for the demand side:

$$\dot{L}_f = L_f\left(\prod_f^e - \overline{\prod}_{i,f}\right) \qquad (10)$$

$$\dot{L}_i = L_i\left(\prod_i^e - \overline{\prod}_{f,i}\right) \qquad (11)$$

where $\overline{\prod}_{f,i}$: represents the average payoff for firms. $\eta_i + \eta_f = 1$ we obtain the following dynamic replicator in the simplex form

$$\dot{\eta}_f = \eta_f \eta_i \{\theta[y_f - (1+\gamma)w_f] - (1-\theta)[y_i - w_i - \rho m]\} \qquad (12)$$

$$\dot{\eta}_i = \eta_i \eta_f \{(1-\theta)[y_i - w_i - \rho m] - \theta[y_f - (1+\gamma)w_f]\} \qquad (13)$$

3.3. The Equilibrium in the labour market

$$\dot{w}_f = \phi[\eta_f - n_f] \qquad (14)$$

$$\dot{w}_i = \phi[\eta_i - n_i] \qquad (15)$$

where $\phi > 0$ measures the responsiveness of changes in wage rates due to the differences in labour demand and supply in each of the sectors. Steady state: $\dot{n}_f = \dot{n}_i = \dot{\eta}_f = \dot{\eta}_i = \dot{w}_f = \dot{w}_i = 0$.

Markets clear: $\eta_f = n_f$ and $\eta_i = n_i$

Three possible equilibriums: (i) $\eta_f = n_f = 0$, (ii) $\eta_f = n_f = 1$, or (iii) $0 < \eta_f = n_f < 1$,

Production in (i): $Y_i = y_i N$

Production in (ii): $Y_{ii} = y_f N$.

Production in (iii): $Y_{iii} = \{n_f^* \eta_f^* y_f + n_i^* \eta_i^* y_i\} N$,

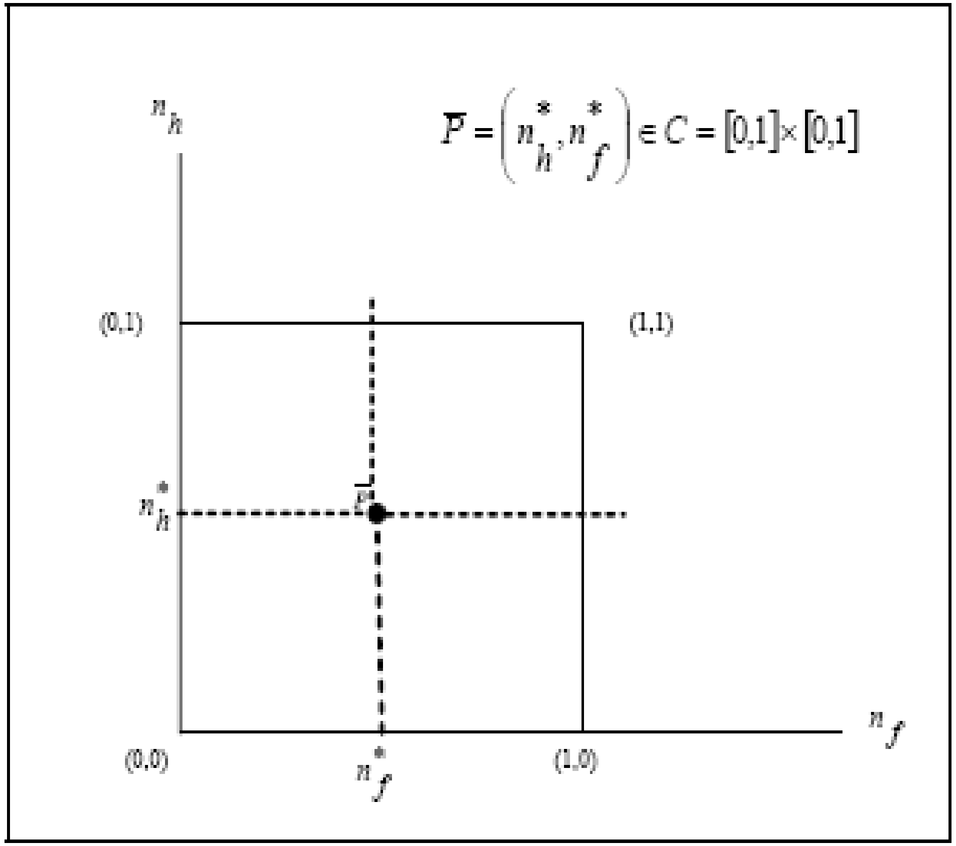


Figure 2a. $\mathcal{C}$-square: equilibria points $(0,0), (0,1), (1,0), (1,1)$ and $\bar{P}$.

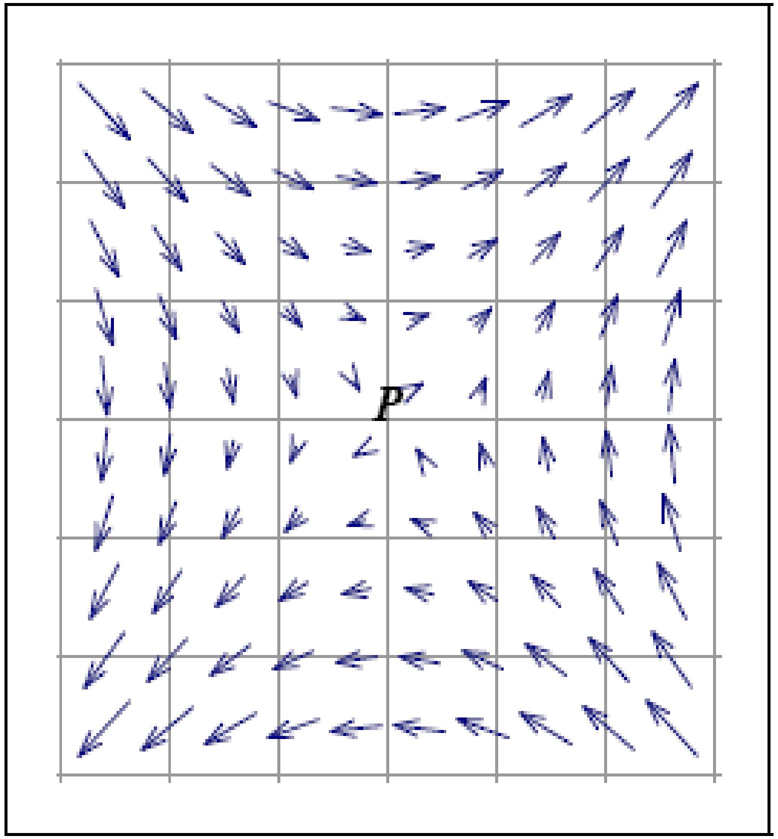


Figure 2b. Vector field of the $\mathcal{C}$-square.

Results (cont.)

- Mixed Equilibrium:

a) Formality and informality coexist;

b) The final outcome is affected by the parameters of the model such as the taxation, amount of fine and probability of being caught in the informal sector;

c) Can be found in some countries of Latin America and OECD.

5. Concluding Remarks

- An increase in taxes, despite producing a direct States' profit increase, results in the diminution of agents' incentives of working within legality.
- By accepting a decrease in taxes, the State may obtain compensation in its income due to the increase in the number of agents that choose legal operation.
- This is shown to be the best policy since the Government maximizes its profits in order to provide the greatest quantity of public services possible and also stimulates economic agents to choose legal operation.

5. Concluding Remarks (cont.)

- Future research:

a) extension of this approach considering unemployment and thus non-flexible wages.

b) an equilibrium analysis of the model.

c) The inclusion of heterogeneous agents, risk averse workers and a joint probability of being caught due to the size of the informal sector

5. References

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