Nash equilibrium: his two interpretations and the relationship with Cournot

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Outline

1. Motivation
2. Equilibrium points
3. Reaching equilibrium

1. Motivation

- Nash awarded Nobel Prize in Economics for
  (i) distinguishing between cooperative and non-cooperative games (Nash 1950c, 1951)
  (ii) and providing equilibrium concept for non-cooperative games (Nash 1950b, 1950c)
- Equilibrium concept formulated by Cournot (1838) in specific application: Hurwicz (1953:402): "the Nash solution, when it is applied to the classic oligopoly problem (the mineral water example, for instance) essentially corresponds to the so-called 'Cournot solution'".

- key concept that players select mutual best responses in equilibrium is in Cournot (1838), so might call it "Cournot-Nash equilibrium"
- However, Cournot's dynamics not accepted by economics profession (see, e.g., Leonard 1994)
- Another fundamental contribution by Nash (1950a, 1950c) is two interpretations of how equilibrium is reached

2. Equilibrium points

Any *n*-tuple of strategies, one for each player, may be regarded as a point in the product space obtained by multiplying the *n* strategy spaces of the players. One such *n*-tuple counters another if the strategy of each player in the countering *n*-tuple yields the highest obtainable expectation for its player against the *n* - 1 strategies of the other players in the countered n-tuple. tuple. A self-countering *n*-tuple is called an equilibrium point. (Nash 1950b).

Cournot's solution for duopoly

Two springs of mineral water: each proprietor seeks to maximize profits:

$$D_i \cdot p(D_1 + D_2), \quad i = 1,2.$$

To compute equilibrium, each propietor takes the production of the other as given in first order conditions for an interior maximum:

$$p(D_1 + D_2) + D_i \cdot p'(D_1 + D_2) = 0, \quad i = 1,2,$$

- Let demand functions be: $p = a - (D_1 + D_2)$
- Optimal response of each proprietor (Cournot):

$$D_1^* = R_1(D_2) = \frac{a-D_2}{2},$$

$$D_2^* = R_2(D_1) = \frac{a-D_1}{2}.$$

- Fixed point (Nash):

$$D_1^{**} = R_1(D_2^*) = R_1\big(R_2(D_1)\big) = \frac{a}{4} + \frac{D_1}{4}.$$

Figure 1. Best response functions (Cournot)

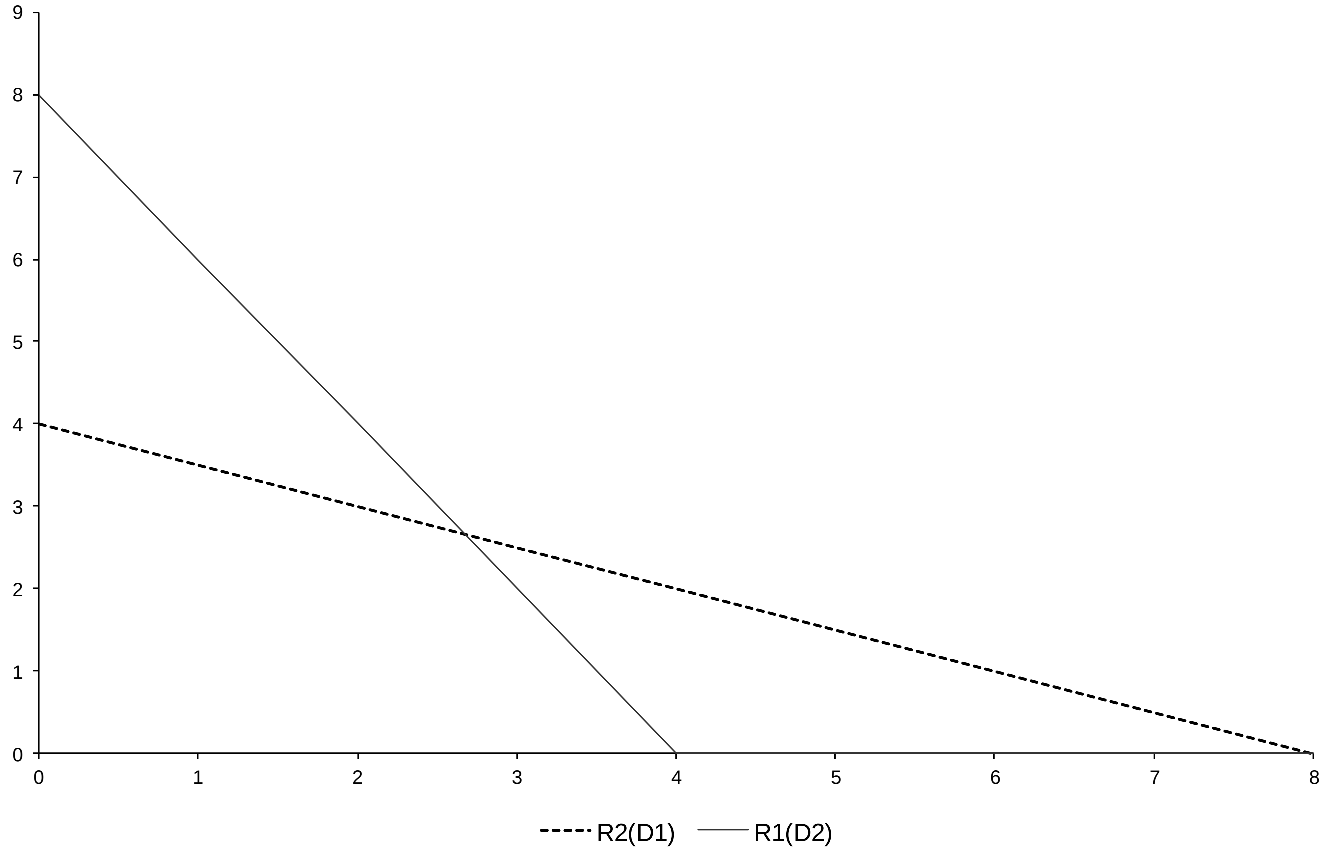
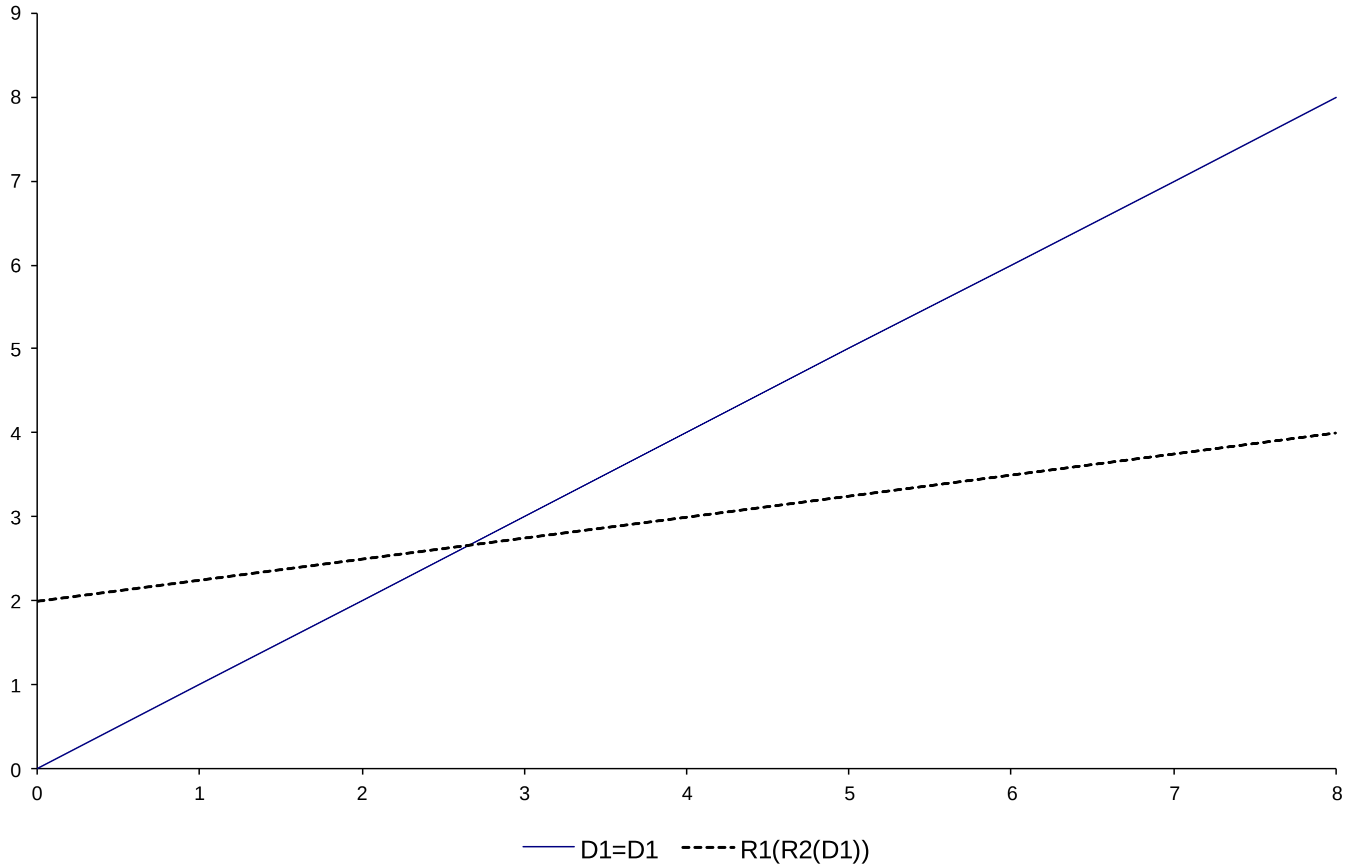


Figure 2. Fixed points (Nash)



Strategic independence of decisions

- Cournot contrasts his solution to collusive solution
- Cournot gives a strategic foundation to his non-cooperative approach, what we would today call "individual rationality" beyond what is collectively convenient
- Both proprietors have benefit from colluding to produce less and raise prices, but they have a "*temporary benefit*" from deviating (Cournot 1838).

- Can represent as a prisoner's dilemma with normal form introduced by Borel (1921) and von Neumann (1928)
- Consider only two production levels

Table 1. Normal form for duopoly

	Collusion ($D_2 = 2$)	Duopoly ($D_2 = 2.67$)
Collusion ($D_1 = 2$)	8, 8	6.7, $\underline{8.9}$
Duopoly ($D_1 = 2.67$)	$\underline{8.9}$, 6.7	$\underline{7.1}$, $\underline{7.1}$

Specific application vs. general formulation

- Main objection (Myerson 1999): not confound application of a methodology with its general formulation
- Myerson recognizes that Cournot was aware that he was developing a general method
- Generality of Cournot’s contribution can be seen from another viewpoint: Cournot’s achievement was to model economic decisions as optimization problems using the tools of calculus.

- But Cournot's specific application made it hard for the readers to distinguish between the general methodology and the specific model (e.g., Bertrand solution: same equilibrium concept for different model)

- Nash (1950b) extends to mixed strategies and von Neumann-Morgestern utility function instead of monetary payoffs, with discrete number of strategies (Cournot considered a continuum of strategies)

3. Reaching equilibrium

- Hotelling (1929) used equilibrium analysis: spatial model that Downs (1957) adopted for political competition
- But Cournot dynamics not generally accepted by the economics profession (Leonard 1994)
- Fellner (1949): behavior of duopolists on reaction curves not rational outside of equilibrium because they do not anticipate what other would do
- Since behavior on reaction curves implausibly myopic, reject equilibrium point too (Fellner 1949).

Nash's idealizing & rationalistic interpretation

- If there is unique solution, players can use it to solve the model: "we need to assume that the players know the full structure of the game in order to be able to deduce the prediction for themselves" (Nash 1950c: 23), but this was not published
- Nash adds that players do not have incentive to act out of conformity with prediction (appears Luce and Raiffa 1957)
- This solves objections to Cournot's dynamics by applying *rational expectations*
- Nash (1950a): unique solution in bargaining game which players use to derive "rational expectations"

Nash’s mass-action interpretation

- If players do not know the structure of the game, adjust mixed strategies putting more weight on pure strategies that give higher payoffs
- Idea ignored, but developed by evolutionary game theory in the 1970s (Maynard Smith): evolutionary stable strategies
- There is anticipation in Cournot (1838: chap. 4): monopolist does not know demand curve but can use elasticity of demand to adjust production
- This idea can be applied to the duopoly model: producers increase production when demand elastic, reduce it when inelastic

- Cournot (1838: chapter 7) presented a hybrid version: implicitly assumes that the players know the demand curve, can model it thus:

$$D^{*}_{i,t} = R_i\left(D_{-i,t-1}\right).$$

- This is now called "best-response dynamics": learning is faster than in replicator dynamics used in evolutionary game theory
- Nash's two interpretations of equilibrium allowed answering long-standing objections to the analysis in Cournot (1838)

References

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