Nash equilibrium: his two interpretations and the relationship with Cournot

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6 June 2016
Outline

1. Motivation
2. Equilibrium points
3. Reaching equilibrium
1. Motivation

- Nash awarded Nobel Prize in Economics for (i) distinguishing between cooperative and non-cooperative games (Nash 1950c, 1951) (ii) and providing equilibrium concept for non-cooperative games (Nash 1950b, 1950c)

- Equilibrium concept formulated by Cournot (1838) in specific application: Hurwicz (1953:402): “the Nash solution, when it is applied to the classic oligopoly problem (the mineral water example, for instance) essentially corresponds to the so-called ‘Cournot solution’”.

- key concept that players select mutual best responses in equilibrium is in Cournot (1838), so might call it “Cournot-Nash equilibrium”
- However, Cournot’s dynamics not accepted by economics profession (see, e.g., Leonard 1994)
- Another fundamental contribution by Nash (1950a, 1950c) is two interpretations of how equilibrium is reached
2. Equilibrium points

Any \( n \)-tuple of strategies, one for each player, may be regarded as a point in the product space obtained by multiplying the \( n \) strategy spaces of the players. One such \( n \)-tuple counters another if the strategy of each player in the countering \( n \)-tuple yields the highest obtainable expectation for its player against the \( n - 1 \) strategies of the other players in the countered \( n \)-tuple. A self-countering \( n \)-tuple is called an equilibrium point. (Nash 1950b).
Cournot’s solution for duopoly

Two springs of mineral water: each proprietor seeks to maximize profits:

\[ D_i \cdot p(D_1 + D_2), \quad i = 1,2. \]

To compute equilibrium, each proprietor takes the production of the other as given in first order conditions for an interior maximum:

\[ p(D_1 + D_2) + D_i \cdot p'(D_1 + D_2) = 0, \quad i = 1,2, \]
- Let demand functions be: \( p = a - (D_1 + D_2) \)
- Optimal response of each proprietor (Cournot):

\[
D_1^* = R_1(D_2) = \frac{a-D_2}{2},
\]
\[
D_2^* = R_2(D_1) = \frac{a-D_1}{2}.
\]

- Fixed point (Nash):

\[
D_1^{**} = R_1(D_2^*) = R_1(R_2(D_1)) = \frac{a}{4} + \frac{D_1}{4}.
\]
Figure 1. Best response functions (Cournot)
Figure 2. Fixed points (Nash)
Strategic independence of decisions

- Cournot contrasts his solution to collusive solution
- Cournot gives a strategic foundation to his non-cooperative approach, what we would today call “individual rationality” beyond what is collectively convenient
- Both proprietors have benefit from colluding to produce less and raise prices, but they have a “temporary benefit” from deviating (Cournot 1838).
- Can represent as a prisoner’s dilemma with normal form introduced by Borel (1921) and von Neumann (1928)
- Consider only two production levels

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Specific application vs. general formulation

- Main objection (Myerson 1999): not confound application of a methodology with its general formulation
- Myerson recognizes that Cournot was aware that he was developing a general method
- Generality of Cournot’s contribution can be seen from another viewpoint: Cournot’s achievement was to model economic decisions as optimization problems using the tools of calculus.
- But Cournot’s specific application made it hard for the readers to distinguish between the general methodology and the specific model (e.g., Bertrand solution: same equilibrium concept for different model)

- Nash (1950b) extends to mixed strategies and von Neumann-Morgestern utility function instead of monetary payoffs, with discrete number of strategies (Cournot considered a continuum of strategies)
3. Reaching equilibrium

- Hotelling (1929) used equilibrium analysis: spatial model that Downs (1957) adopted for political competition
- But Cournot dynamics not generally accepted by the economics profession (Leonard 1994)
- Fellner (1949): behavior of duopolists on reaction curves not rational outside of equilibrium because they do not anticipate what other would do
- Since behavior on reaction curves implausibly myopic, reject equilibrium point too (Fellner 1949).
Nash’s idealizing & rationalistic interpretation

- If there is unique solution, players can use it to solve the model: “we need to assume that the players know the full structure of the game in order to be able to deduce the prediction for themselves” (Nash 1950c: 23), but this was not published

- Nash adds that players do not have incentive to act out of conformity with prediction (appears Luce and Raiffa 1957)

- This solves objections to Cournot’s dynamics by applying rational expectations

- Nash (1950a): unique solution in bargaining game which players use to derive “rational expectations”
Nash’s mass-action interpretation
- If players do not know the structure of the game, adjust mixed strategies putting more weight on pure strategies that give higher payoffs
- Idea ignored, but developed by evolutionary game theory in the 1970s (Maynard Smith): evolutionary stable strategies
- There is anticipation in Cournot (1838: chap. 4): monopolist does not know demand curve but can use elasticity of demand to adjust production
- This idea can be applied to the duopoly model: producers increase production when demand elastic, reduce it when inelastic
- Cournot (1838: chapter 7) presented a hybrid version: implicitly assumes that the players know the demand curve, can model it thus:

\[ D_{i,t}^* = R_i(D_{-i,t-1}). \]

- This is now called “best-response dynamics”: learning is faster than in replicator dynamics used in evolutionary game theory.
- Nash’s two interpretations of equilibrium allowed answering long-standing objections to the analysis in Cournot (1838)
References


