Credible signals: A refinement of perfect Bayesian equilibria

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1. Tuesday: recap
2. Motivation of refinement
3. Definitions: intuitive criterion and self-selection condition
4. Application: two signaling games
5. Closing remarks

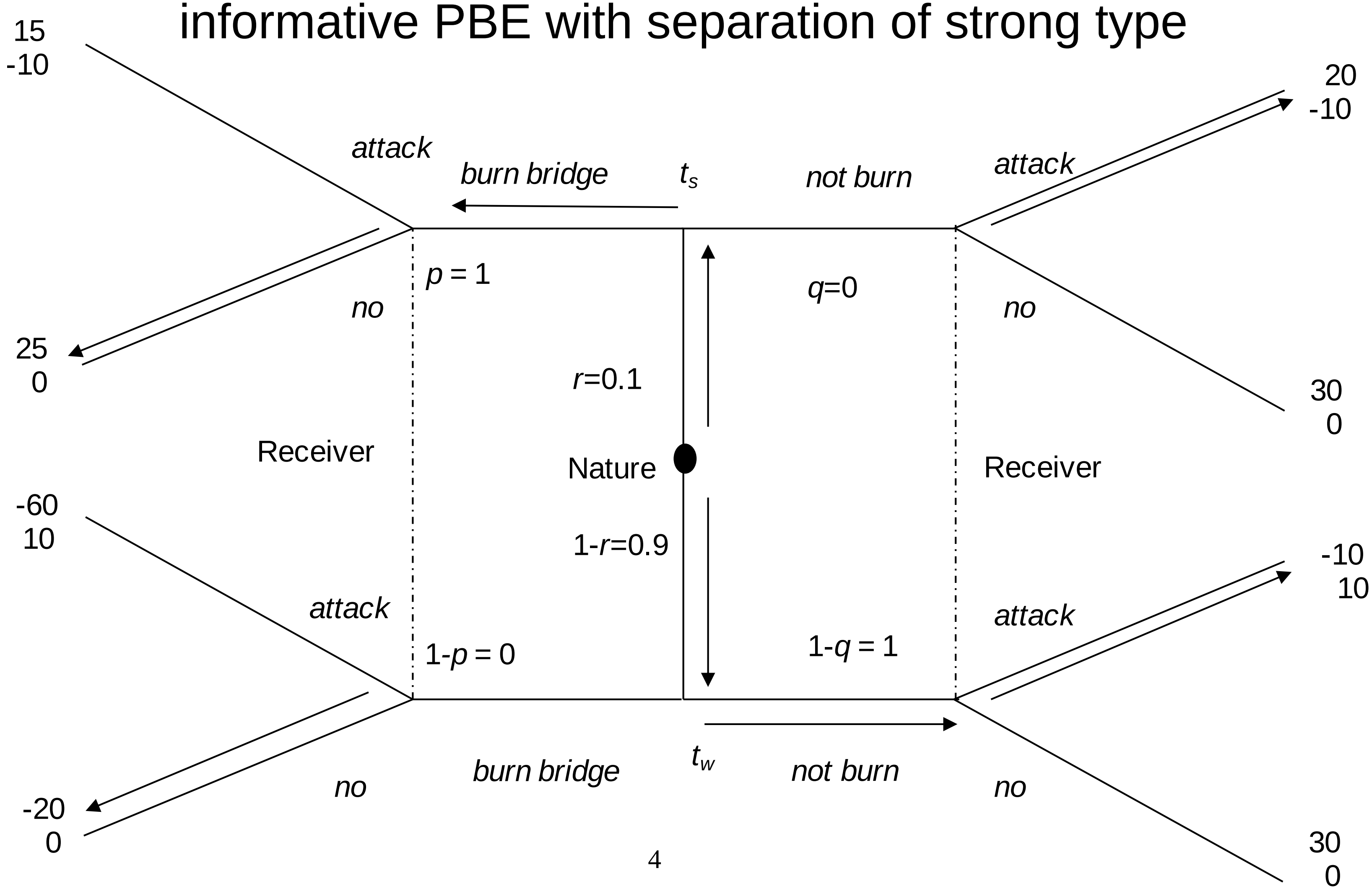
1. Tuesday: recap

Equilibrium concepts

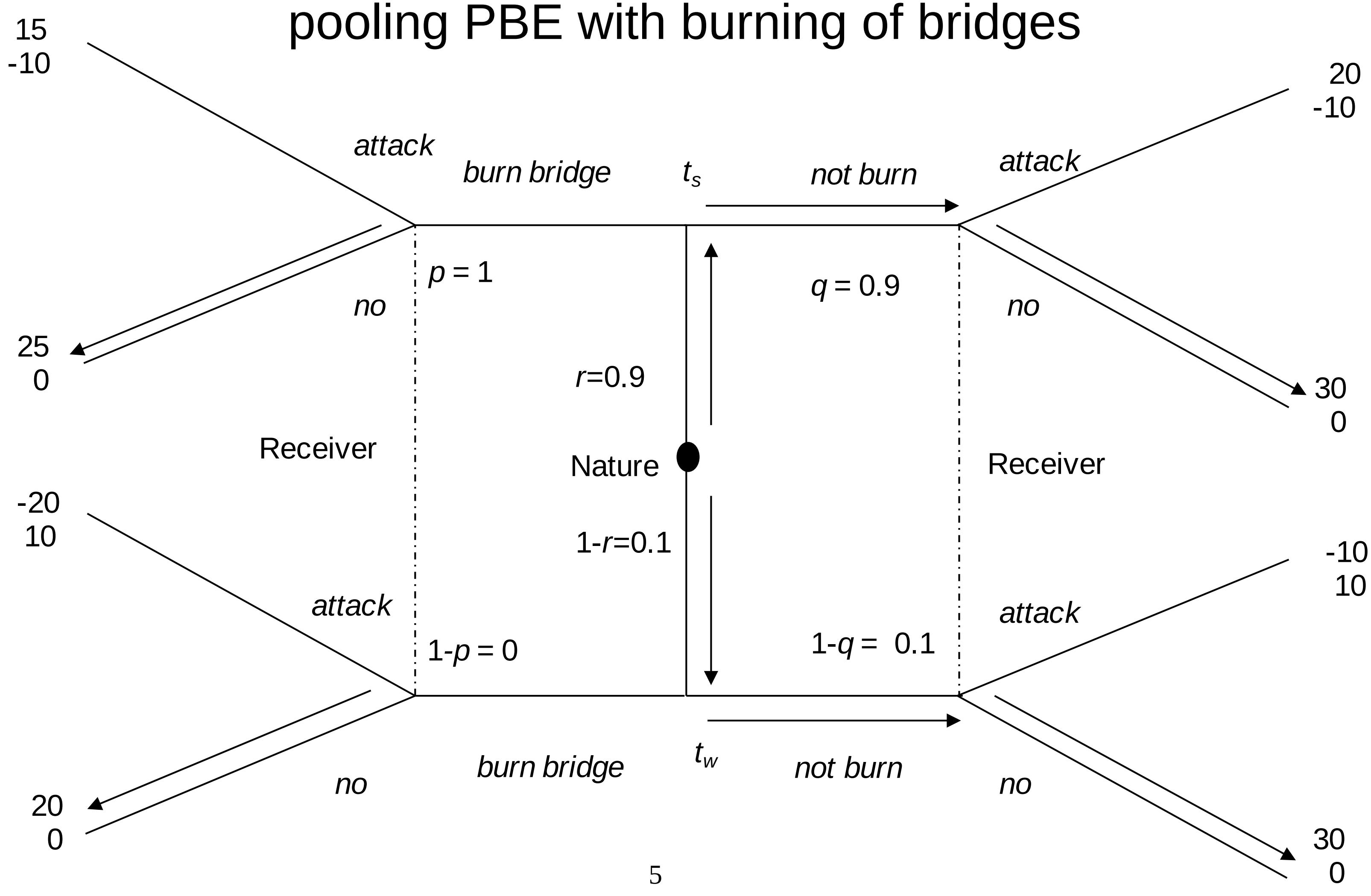
		Information on types	
		Complete	Incomplete
Information on actions	Imperfect	Nash eq.	Bayes Nash eq.
	Perfect	Subgame-perfect Nash eq.	Perfect Bayesian eq. (PBE)

- Refinements require beliefs off the equilibrium path to be specified, and players to pick a best response to those beliefs

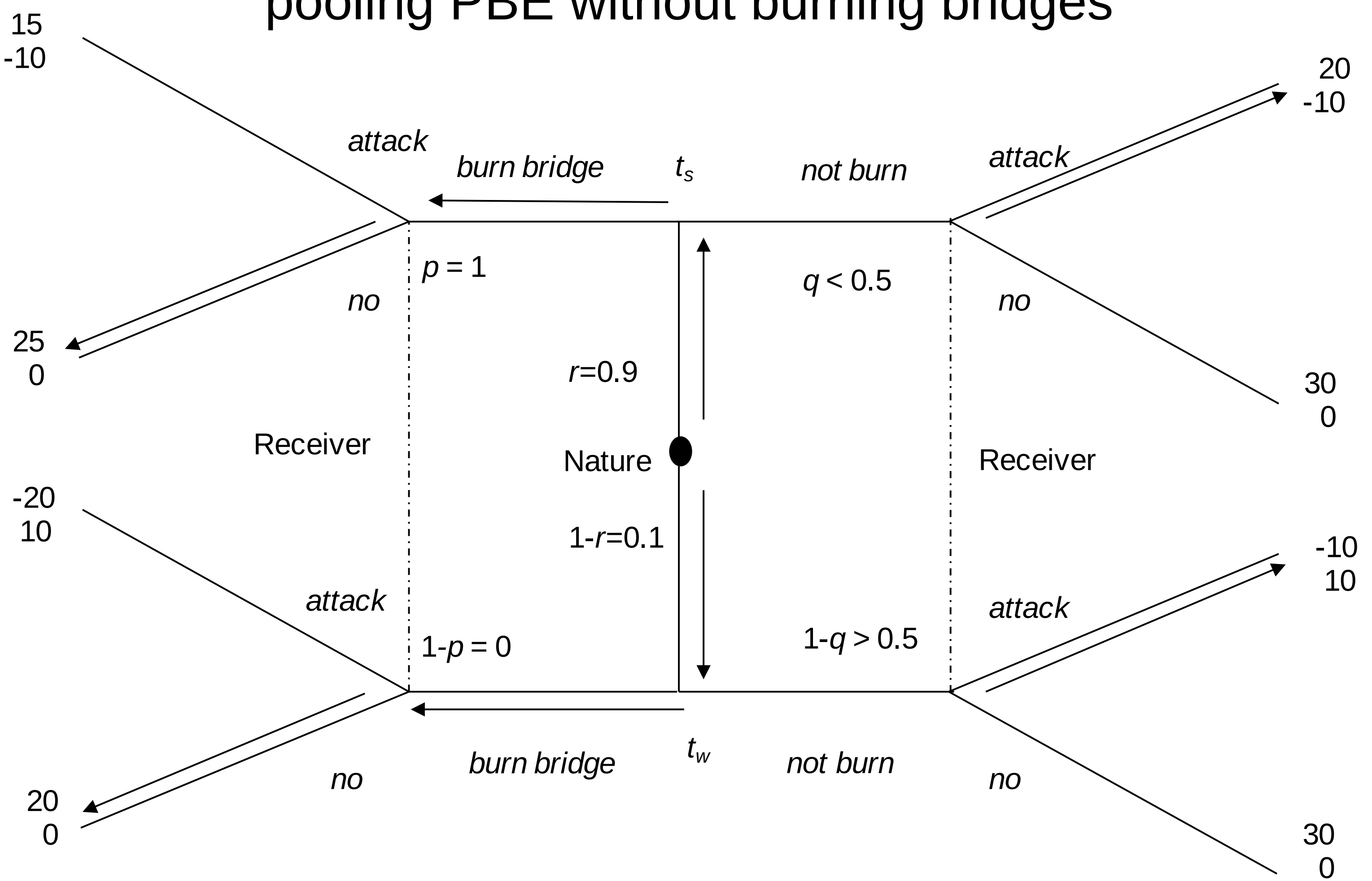
Signaling game 1 (weak type has high signaling costs): informative PBE with separation of strong type



Signaling game 2 (weak type has low signaling costs): pooling PBE with burning of bridges



Signaling game 2 (weak type has low signaling costs): pooling PBE without burning bridges



- In signaling game 1, priors are that weak type more likely, and burning bridges is very costly for weak type; since burning bridges is too costly for weak type to mimic, strong type can separate out (this is Schelling story)
- In signalig game 2, priors are that strong type is more likely, and burning bridges is not too costly for weak type; since burning bridges is cheap for weak type, only have pooling equilibria
- But pooling equilibria where both type burns bridges is a bit strange: have to burn bridges so priors do not change, where do out-of-equilibrium beliefs come from? See how to eliminate this pooling equilibrium.

2. Motivation

- Milgrom-Roberts (1982) entry deterrence game: pooling and separating equilibria for same parameter values. Since no entry in pooling equilibria, why would low cost incumbent pick costly signal rather than monopoly output if priors deter entry?
- Similar for signaling game 2 based on Schelling: if priors deter attack, why need of burning bridges to keep priors (i.e., why second pooling equilibrium)?
- This motivates refinement: signals are voluntary, so senders pick a signal if it improves payoffs in relation to Bayes-Nash equilibrium given by priors

Intuitive criterion:

- Allows to restrict beliefs off the equilibrium path
- However, it does not eliminate either of these equilibria
- Furthermore, it can discard equilibria no sender would want to discard: example is Spence (1973) job-market model

Alternative proposal: self- selection criterion

- analysis in terms of equilibrium outcomes: for a signal to be a credible deviation, it has to be part of an equilibrium
- Credible signals are then combined with idea that senders self-select equilibrium

3. Definitions of intuitive criterion and self-selection condition

- Games with incomplete information where sender can send a signal about its type to receiver.
- Consider discrete number I of types $w_i \in \mathrm{W}$
- Also consider discrete number J of signals $m_j \in \mathrm{M}$
- receiver only observes m_j, not w_i
- In some applications these may be continuous

Timing of signaling game

(i) Sender type $w_i \in$ W drawn from commonly known distribution $p(w) = (p(w_1), \dots, p(w_I))$

(ii) Sender S observes its type w_i and picks an action: signal or message $m_j \in$ M.

(iii) Receiver R observes the signal m_j, but not the sender's type w_i, forming beliefs $\mu(m_j)$ and picking an action $a_k \in$ A in response.

(iv) Payoffs given by $v^S(w_i, m_j, a_k)$ and $v^R(w_i, m_j, a_k)$.

Strategies and beliefs $(\sigma^S(w), \sigma^R(m), \mu(m))$:

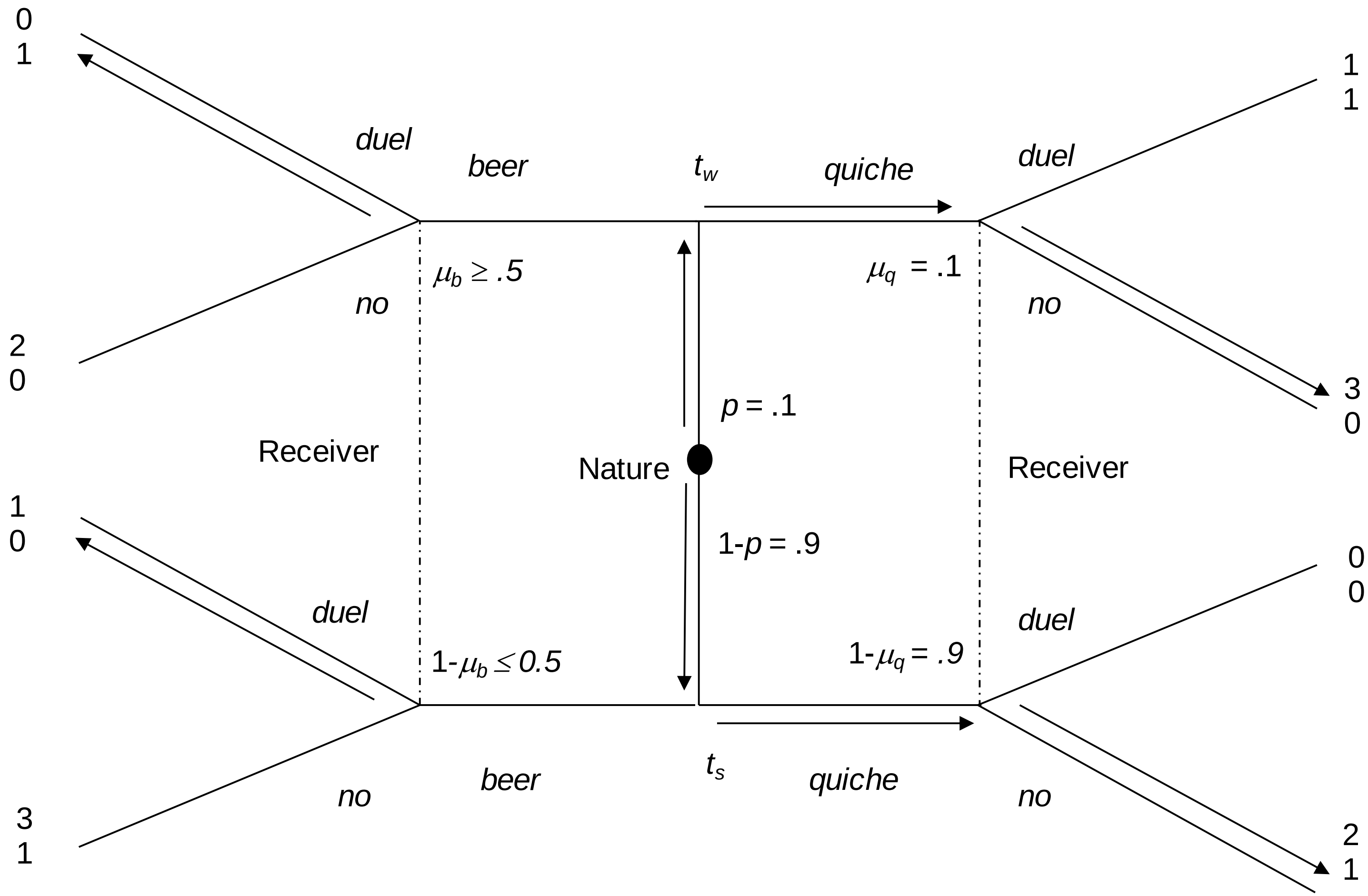
- strategy sender $\sigma^S(w) = (\sigma^S(w_1), \ldots, \sigma^S(w_I))$ is vector probability distributions $\sigma^S(w_i) = (\sigma^S(w_i)(m_1), \ldots, \sigma^S(w_i)(m_J))$;
- strat. receiver $\sigma^R(m) = (\sigma^R(m_1), \ldots, \sigma^R(m_J))$ is vector probability distributions $\sigma^R(m_j) = (\sigma^R(m_j)(a_1), \ldots, \sigma^R(m_j)(a_K))$;
- beliefs receiver $\mu = (\mu(m_1), \ldots, \mu(m_J))$ is vector probability distribution $\mu(m_j) = \left(\mu(m_j)(w_1), \ldots, \mu(m_j)(w_I)\right)$.

Cho-Kreps intuitive criterion

DEFINITION (equilibrium dominance): a strategy off the equilibrium path is dominated in equilibrium for a given type if the highest possible payoff from deviating to that strategy is lower than the equilibrium payoff.

DEFINITION (intuitive criterion): if a strategy that is off the equilibrium path is not dominated in equilibrium for a single type (i.e., it is dominated in equilibrium for all other types but this one), then beliefs should only put positive weight on that type.

Beer and quiche game: pooling on quiche



$\mu_b \equiv \mu(t_w \mid beer),\ \mu_q \equiv \mu(t_w \mid quiche)$

- The intuitive criterion works well here in the beer-quiche game: discards pooling on quiche, only pooling on beer is left.
- But intuitive criterion does not solve multiplicity of equilibria problem in the Schelling game of burning bridges, or in the Milgrom and Roberts entry-deterrance game (and it introduces odd features in Spence job market model: siren calls)
- I propose an alternative refinement to restrict beliefs off the equilibrium path. Key idea is credible deviations.

DEFINITION 2 (credible deviation). *A deviation* m_j *from a given equilibrium* $(\tilde{\sigma}^S(w), \tilde{\sigma}^R(m), \tilde{\mu}(m))$ *is credible for a sender type* w_i *if that deviation is part of an alternative equilibrium* $(\tilde{\tilde{\sigma}}^S(w), \tilde{\tilde{\sigma}}^R(m), \tilde{\tilde{\mu}}(m))$ *and the payoff* $v^S(w_i, \tilde{\sigma}^S(w), \tilde{\sigma}^R(m))$ *from the current equilibrium is not larger than the payoff* $v^S\big(w_i, \tilde{\tilde{\sigma}}^S(w), \tilde{\tilde{\sigma}}^R(m)\big)$ *from the deviation to* m_j *in the alternative equilibrium.*

DEFINITION 3 Self-selection condition:

(i) If a deviation is not credible for certain player types, beliefs should place zero probability on these types, unless the same holds for all other types in W*, in which case the restriction is moot. (ii) If credible deviations lead all sender types to pick an alternative equilibrium, beliefs are determined by the alternative equilibrium.(iii) If credible deviations lead to a cycle among equilibria, they impose no restrictions on beliefs.*

This is similar to weak announcement-proofness in Mathews, Okuno-Fujiwara and Postlewaite, but applies to all signaling games, not only cheap-talk

4. Two signaling games: self-selection condition

- Milgrom and Roberts (1982) entry deterrence game: with intuitive criterion there are multiple equilibria; self-selection condition either selects pooling equilibrium or separating equilibrium

- Spence (1973) job-market model: selects separating equilibrium only for some parameter values; for others, selects pooling equilibrium

- Note: in the Cho and Kreps (1987) beer and quiche game, selects same equilibrium as intuitive criterion

A. Entry deterrence game: incumbent and potential entrant

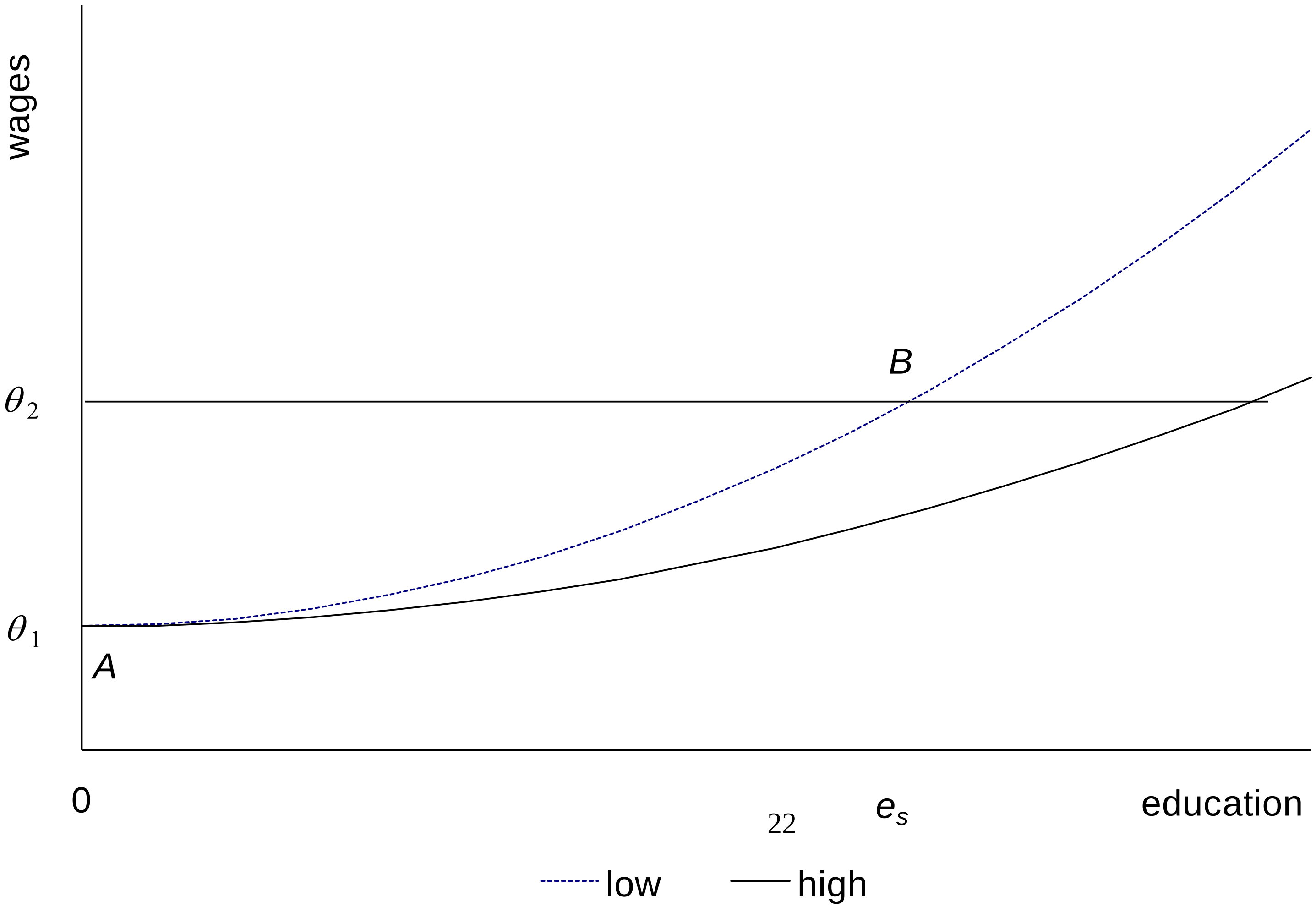
- demand function is $p(q) = 9 - q$.
- incumbent i: low marginal cost $c_L = 1$ with probability λ, high marginal cost $c_H = 3$ with probability $1 - \lambda$.
- potential entrant e has a known marginal cost of 3 and incurs a fixed cost of 3 if it enters.
- both firms maximize profits π.

Timing

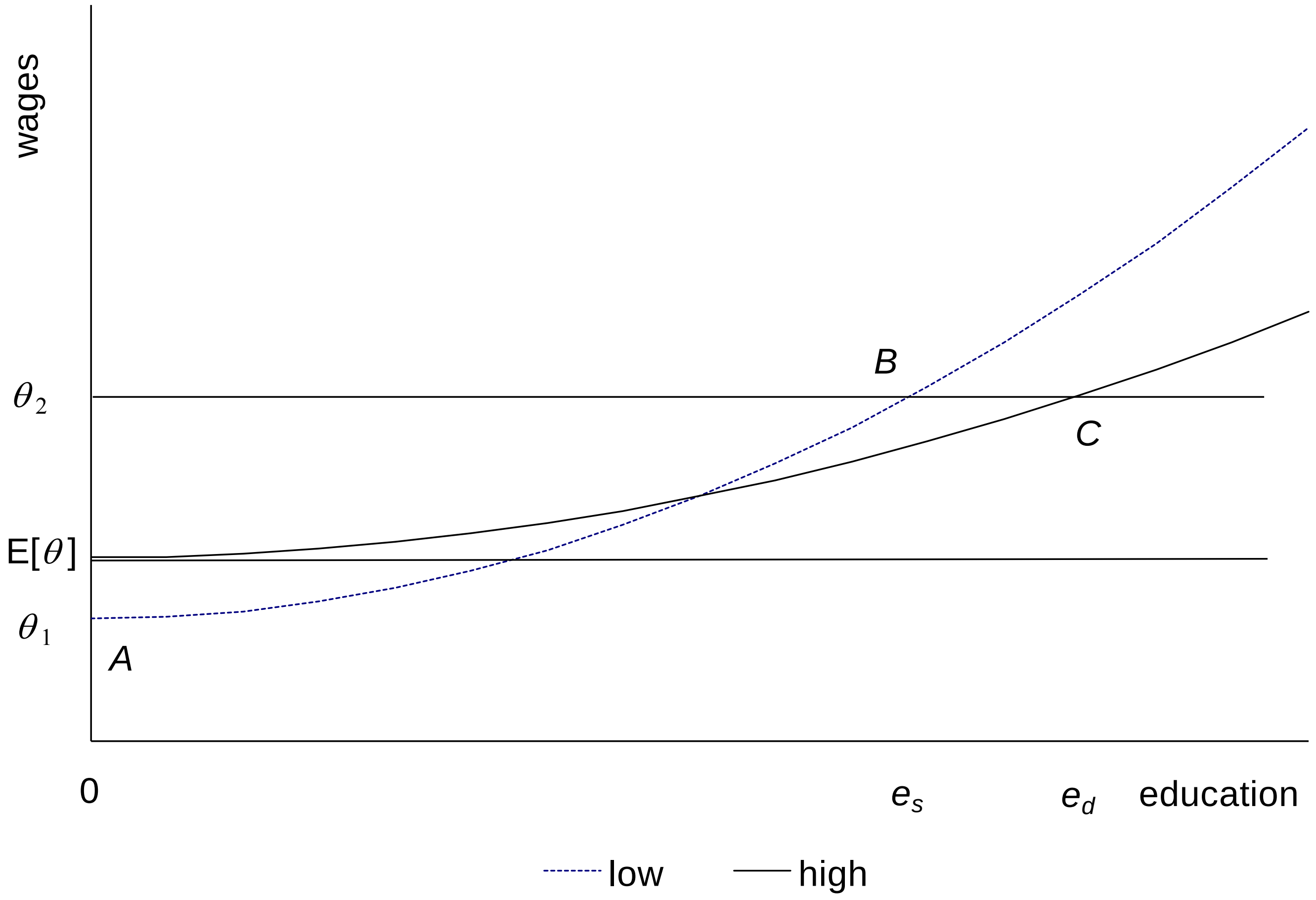
- there are two periods. There is no discounting.
- in the first period, the incumbent is a monopolist. The entrant observes the price that the incumbent charges.
- in the second period, the entrant decides whether to enter or not. After that, the incumbent's marginal costs are revealed.
- if the entrant decided to enter, the firms engage in Cournot competition, otherwise the incumbent acts like a monopolist.

- with no entry in second period: $p_2 = 5$ with low-cost incumbent; $p_2 = 6$ with high-cost incumbent.
- Separating equilibria with $2.79 \leq p_l \leq 3.76$
- For probability low cost $\lambda \geq \frac{9}{20}$, pooling equilibria also exist: $3.76 \leq p_p \leq 7.21$
- Intuitive criterion does not eliminate all pooling equilibria
- Self-selection condition: if there is pooling equilibrium, there is no separating equilibrium,
- Selects either pooling equilibrium $p_p = 5$
  or most efficient separating equilibrium

B. Spence job market model: efficient separating equilibrium



Job market model: maximum deviation from pooling eq.



5. Final Remarks

- Intuitive criterion goes a bridge too far? Can work as a siren call, so propose two modifications:
(i) Credible signals instead of equilibrium dominance: look at payoffs of deviations that are potentially part of another equilibrium
(ii) senders self-select equilibrium: since signal is voluntary, only send informative signal if some sender type better off than with priors; on the other hand, if all sender types better off, informative signals are to be expected (similar to weak announcement-proofness)

- Note on self-selection condition: only used it to restrict beliefs that were off the equilibrium path and belonged to an alternative PBE
- Can extend the self-selection condition to restrict beliefs on the equilibrium path: this is more controversial than restricting beliefs off the equilibrium path
- This does not make a difference in the games we saw, but it could make a difference in games such as a discrete version of Spence where only two signals are available (zero education or the separating signal): in inefficient separating equilibrium, the deviation to zero education is on equilibrium path

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