Taxing Cognitive and Non-Cognitive Skills

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Abstract

In this paper we investigate the role of cognitive and non-cognitive skills on optimal taxation. We consider cognitive skills associated with schooling and human capital accumulation while non-cognitive skills are related to the trade-off between today's decision versus tomorrow's. In our paper that specifically concerns consumption of health/unhealthy food, studying/leisure/working and savings. We show that the policy package that implements the social optimum contains subsidies directed to wealth, health and human capital stocks and not taxes on current (flow) allocations. We further explore how a paternalistic optimal policy must take also into account redistributive reasons when dealing with potential interactions of cognitive and non-cognitive skills.

Keywords: Paternalism; Optimal Taxation; Education; Health.

JEL Classification: D62, H21, H31, H23, I18.

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1 Introduction

The main goal of this paper is to investigate the role of cognitive and non-cognitive skills on optimal taxation. We consider cognitive skills associated with schooling and human capital accumulation while non-cognitive skills are related to the impatience associated to the individual when calculating the trade-off between today's decision versus tomorrow's. In our paper that specifically concerns consumption of health/unhealthy food, studying/leisure/working and savings and future payoffs of those decisions. We assume that the instantaneous utility depends on the current consumption of healthy and unhealthy goods, current health status (stock of health capital) and leisure. An agent's stock of health capital, in turn, depends on all past consumption of the unhealthy good. Although the current human capital stock does not affect agents' instantaneous utility directly, his/her current and past decisions regarding schooling affect human capital accumulation and, consequently, leisure-labor-school choices. Therefore, in our model, the externality that the individual's current self imposes on his/her future selves is a two-dimension stock-externality, which is in line with standard models of human capital and in health economics (e.g., Grossman (1972, 2000)). In the Ramsey optimal taxation tradition, we show that the policy package that implements the social optimum contains subsidies directed to wealth and health and human, either separately or jointly through their effect on an agent's labor earnings. The optimal policy set does not contain a tax on the unhealthy consumption or a subsidy on the flow of resources spent to improve an individual's health. We further explore how a paternalistic optimal policy must not only take into account agents' self-control problems but also potential interactions of such (lack of) skills and cognitive skills.

Knowledge about human behavior from psychology and sociology has enhanced the field of economics of education and health. There is now extensive evidence that cognitive skills (as measured by achievement tests) and soft skills (personality traits not adequately measured by achievement tests) are equally important drivers of later economic outcomes (Shoda et al. (1990), Golsteyn et al. (2014), Koch et al. (2015), Courtemanche et al. (2015)). In our model, cognitive skills refers to an agent's ability to accumulate more human capital at a low leisure cost, i.e., how schooling effort translates into human capital. Different individuals face different costs of acquiring human capital, measured by the effective time cost (in terms of leisure) per unit of time devoted to human capital formation. For each unit of time allocated to the accumulation of human capital, an individual with less cognitive skills sacrifices more time at schooling.

In our setup, non-cognitive skills are associated with future decisions (and their consequences) that include consumption of unhealthy food, savings and labor-school-leisure choice. Our modeling approach to those skills also called as soft skills (Koch et al. (2015)) follows an extensive literature on present-bias and quasi-hyperbolic discounting (see, for instance, Laibson (1997), O'Donoghue and Rabin (1999) and Gruber and Koszegi (2004)). Regarding health status, we follow O'Donoghue and Rabin (2003, 2006) and Aronsson and Thunstrom (2008). There is a paternalistic motive for optimal taxation when self-control problems caused by quasi-hyperbolic discounting may lead to excessive consumption of unhealthy food. We assume that the instantaneous utility increases with the current consumption of an unhealthy goods and decreases with the consumption of the same unhealthy good in the previous period. Through the law of motion of health status, the latter captures future health consequences of the current consumption. With a self-control problem, this mechanism also implies that the individual's current self imposes a flow-externality on his/her future selves, as the future welfare effects associated with the current instantaneous consumption are not fully internalized (Aronsson and Thunstrom (2008)).

Redistribution and agents misperceptions regarding future consequences of their present decisions have motivated government interventions through public policy. When the government and consumers differ regarding their preferences, the government might wish to influence private agents' choices using the policy instruments available. Excessive consumption of goods with negative health effects (such as unhealthy food, cigarettes and alcohol) by consumers with self-control problems is an example of a situation where consumers do not fully take into account the future negative effects caused by the consumption of such goods. Another important example is schooling decision and the positive future consequences for a worker's earnings. Misperception of the complementarity of education and health and their effect on agents' productivity and the economy's output might also lead to sub-optimal allocations, justifying government intervention.

Tax policies in the context of present-bias and self-control problems have been considered, for example, by Laibson (1997), O'Donoghue and Rabin (2003, 2006) and Gruber and Koszegi (2004). In general, policies of this kind are an example of paternalism, and their purpose is to protect individuals when they act against their own best self-interest. There is a growing literature studying present-biased preferences in different contexts. Optimal sin taxes have been studied by, for instance, O'Donoghue and Rabin (2003, 2006) and Gruber and Koszegi (2004). This literature considers how linear taxes can be used to either prevent over consumption of some goods (e.g., fossil fuels, drugs) or to foster consumption of other goods (e.g., retirement savings). They model an economy where individuals have hyperbolic preferences and differ both in their taste for the sin good and in their degree of time inconsistency. The authors show how (heterogeneity in) time inconsistency affects the optimal (Ramsey) consumption tax policy. Farhi and Gabaix (2015) find that if heterogeneity in behavioral biases is sufficiently high, quantity restrictions on consumption of the "behavioral" good for all agents fare better than linear taxes.[1]

Grossman and Kaestner (1997) and Grossman (2000, 2005), among others, have provided

[1] Aronsson and Granlund (2011) study the provision of a public good in a two-type model under present-biased consumer preferences. The preference for immediate gratification facing the high-ability type weakens the incentive to adjust public provision in response to the self-selection constraint. In Cremer et al. (2012), individuals are myopic and underestimate the effect of the sinful consumption on health and they may acknowledge, in their second period, their mistake or persist in their error. They characterize and compare the first-best and the (linear) second-best taxes when sin-good consumption and health care interact in health production technology. *Aronsson and Thunstrom (2008)* show that subsidies on wealth and health capital can be used to implement a socially optimal resource allocation. See also Lockwood (Working Paper, 2016)

detailed reviews of evidence regarding the education-health gradient along a variety of health measures. Their extensive review suggests that years of formal schooling completed is the most important correlate of good health. This finding emerges whether health levels are measured by mortality rates, morbidity rates, self-evaluation of health status, or physiological indicators of health, and whether the units of observation are individuals or groups. The studies also suggest that schooling is a more important correlate of health than occupation or income.[2] In our model education and health decisions affect an individual's labor earnings and we study the impacts on the individual's decisions of policy instruments such as subsidies on health status and human capital stock. We also investigate the optimality of an income tax under the premise the effects of education and health are materialized in the individuals' earnings and policies should target target both (Edu (2007)).

The paper is divided as follows. Section 2 presents our model and key elements of our economy. In Section 3, we characterize the optimal policies. Section 6 concludes the paper. To the best of our knowledge, this is the first study to consider paternalistic taxation in the context of of health and human capital accumulation, accounting for cognitive and non-cognitive skills.

2 The Model

We consider an economy consisting of $I \times J$ types of individuals indexed by superscript ij, for $i \in [1, I]$, $j \in [1, J]$). Agents are different regarding their cognitive (i) and non-cognitive (j) skills. Cognitive skills refer to an agent's exogenously given endowment of the complementary factors to the schooling process, i.e., skills associated with agent's ability to accumulate more human capital at a low leisure cost. On the other hand, non-cognitive skills are associated with future decisions (and their consequences) that include consumption of unhealthy food, savings and deciding to work o leisure instead of investing in human capital. They are modeled along the lines of the present-bias literature.

The instantaneous utility function facing the ij-type consumer is written

$$u\left(c_t^{ij}, x_t^{ij}, m_t^{ij}\right) \quad + \quad v\left(z_t^{ij}\right) \tag{1}$$

where c_t^{ij} is the consumption of an ordinary (not unhealthy) good, x_t^{ij} the consumption of the unhealthy good, m_t^{ij} the stock of health capital. Leisure z_t^{ij} is defined as a time endowment (normalized to 1) less the time in market work, l_t^{ij}, and hours spent building human capital (studying, training), s_t^{ij}. We assume that functions $u(\cdot)$ and $v(\cdot)$ are increasing in each argument and strictly concave.

[2] In a broad sense, the observed positive correlation between health and schooling may be explained in one of three ways. The first argues that there is a causal relationship that runs from increases in schooling to increases in health. The second holds that the direction of causality runs from better health to more schooling. The third argues that no causal relationship is implied by the correlation; instead, differences in one or more "third variables," such as physical and mental ability and parental characteristics, affect both health and schooling in the same direction.

Let $\zeta^i \in (0,1)$ denote the effective time cost (in terms of leisure) per unit of time devoted to human capital formation, this term captures cognitive skills. That is, for each unit of time that ij-type individual allocates to the accumulation of human capital, she sacrifices a fraction of leisure time equal to $\zeta^i s_t^{ij}$. An agent with high cognitive ability, i.e., low ζ^i, experiences a lower leisure cost of studying - she can accomplish more for each unit of time dedicated to study. This assumption captures the fact that different individuals face different costs of acquiring human capital (Mejia and St-Pierre (2008)). Hence, an individual's leisure is as follows: $z_t^{ij} = 1 - \zeta^i s_t^{ij} - l_t^{ij}$.

An individual chooses among non-mutually exclusive education and labor market options in order to maximize lifetime utility, knowing that current education, consumption habits - healthy and unhealthy food - and labor market decisions affect future wages and health and human capital stocks. We operationalize the concept of present biased preferences by using an approach developed by Phelps and Pollak (1968) and later used by e.g. Laibson (1997) and O'Donoghue and Rabin (2003).

The inter-temporal objective at time t is given by

$$U_t^{ij} = \left[u\left(c_t^{ij}, x_t^{ij}, m_t^{ij}\right) + v\left(z_t^{ij}\right)\right] + \beta^j \sum_{s=t+1}^{\infty} \Theta^{s-t}\left[u\left(c_s^{ij}, x_s^{ij}, m_s^{ij}\right) + v\left(z_t^{ij}\right)\right] \tag{2}$$

where $\Theta^t = 1/(1+\theta)^t$ is a conventional utility discount factor with utility discount rate θ, whereas $\beta^j < 1$ is a time-inconsistent preference for immediate gratification. This last term captures our non-cognitive hypothesis. Following O'Donoghue and Rabin (2003), we assume that the consumer is naive in the sense of not recognizing that the preference for immediate gratification is present also when the future arrives.

We distinguish between an agent's health stock m_t^{ij} and his human capital h_t^{ij}. Time units spent on education/schooling (s_t^{ij}) are interpreted as investment in human capital. Human capital investments require agents to give up labor income or leisure early in the life-cycle in order to generate higher future earnings. Agents derive utility from their health stock (or quality of health), on which x_t^{ij} has a negative effect and health care services e_t^{ij} affect it positively. The agent's human and health capital stocks evolve as follows

$$h_{t+1}^{ij} - (1-\delta_h)\, h_t^{ij} = B\left(s_t^{ij}\right) \tag{3}$$

$$m_{t+1}^{ij} - (1-\delta_m)\, m_t^{ij} = g\left(x_t^{ij}, e_t^{ij}\right) \tag{4}$$

where $B\left(s_t^{ij}\right)$ is an increasing and concave function of the fraction of time invested in human capital formation, s_t^{ij} (i.e., $\partial B\left(s_t^{ij}\right)/\partial s_t^{ij} > 0$) and $g(\cdot)$ is a health production function with the properties $\partial g\left(x_t^{ij}, e_t^{ij}\right)/\partial x_t^{ij} < 0$ and $\partial g\left(x_t^{ij}, e_t^{ij}\right)/\partial e_t^{ij} > 0$.

The household budget constraint is

$$c_t^{ij} + x_t^{ij} + e_t^{ij} + k_{t+1}^{ij} = W_t\left(m_t^{ij} h_t^{ij}\right) l_t^{ij} + (1 + R_t - \delta_k)\, k_t^{ij} \tag{5}$$

where the household holds an asset in the form of physical capital k_t^{ij}. We assume that the agent takes the wage and the interest rates, W_t and R_t, respectively, as exogenous given. The prices of the two consumption goods $\left(c_t^{ij}, x_t^{ij}\right)$ are set equal to one. In period t, the household chooses allocations $\{c_t^{ij}, x_t^{ij}, e_t^{ij}, s_t^{ij}, l_t^{ij}, k_{t+1}^{ij}, m_{t+1}^{ij}, h_{t+1}^{ij}\}$ to maximize the utility function (2) subject to equations (5), (3) and (4), treating the initial physical, health and human capital stocks, k_0^{ij}, m_0^{ij} and h_0^{ij}, as exogenously given. Hence, a ij-type agent problem in Lagrangian form becomes

$$\begin{aligned}
\mathscr{L}_{ij} &= u\left(c_t^{ij}, x_t^{ij}, m_t^{ij}\right) + v\left(z_t^{ij}\right) \\
&+ \beta^j \sum_{s=t+1}^{\infty} \Theta^{s-t}\left[u\left(c_s^{ij}, x_s^{ij}, m_s^{ij}\right) + v\left(z_s^{ij}\right)\right] \\
&+ \lambda_t^{ij}\left[W_t\left(m_t^{ij} h_t^{ij}\right) l_t^{ij} + \left(1 + R_t - \delta_k\right) k_t^{ij} - c_t^{ij} - x_t^{ij} - e_t^{ij} - k_{t+1}^{ij}\right] \\
&+ \beta^j \sum_{s=t+1}^{\infty} \Theta^{s-t} \lambda_s^{ij}\left[W_s\left(m_s^{ij} h_s^{ij}\right) l_s^{ij} + \left(1 + R_s - \delta_k\right) k_s^{ij} - c_s^{ij} - x_s^{ij} - e_s^{ij} - k_{s+1}^{ij}\right] \\
&+ \mu_t^{ij}\left[m_{t+1}^{ij} - \left(1 - \delta_m\right) m_t^{ij} - g\left(x_t^{ij}, e_t^{ij}\right)\right] \\
&+ \beta^j \sum_{s=t+1}^{\infty} \Theta^{s-t} \mu_s^{ij}\left[m_{s+1}^{ij} - \left(1 - \delta_m\right) m_s^{ij} - g\left(x_s^{ij}, e_s^{ij}\right)\right] \\
&+ \xi_t^{ij}\left[h_{t+1}^{ij} - \left(1 - \delta_h\right) h_t^{ij} - B\left(s_t^{ij}\right)\right] \\
&+ \beta^j \sum_{s=t+1}^{\infty} \Theta^{s-t}\left[h_{s+1}^{ij} - \left(1 - \delta_h\right) h_s^{ij} - B\left(s_s^{ij}\right)\right]
\end{aligned} \tag{6}$$

Let $u^{ij}(t) = u\left(c_t^{ij}, x_t^{ij}, m_t^{ij}\right)$ and $u_c^{ij}(t) = \partial u^{ij}(t)/\partial c_t^{ij}$, for an ij-type individual, and likewise for other allocations and functions. Combining the first order conditions for the household, while eliminating the Lagrange multipliers, the necessary conditions for an interior solution of the household's maximization problem are given by

$$u_x^{ij}(t) - u_c^{ij}(t) + u_c^{ij}(t)\frac{g_x^{ij}(t)}{g_e^{ij}(t)} = 0 \tag{7}$$

$$u_c^{ij}(t) - \beta u_c^{ij}(t+1)\left[1 + R_t - \delta_k\right] = 0 \tag{8}$$

$$-\frac{u_c^{ij}(t)}{g_e^{ij}(t)} + \beta^j \Theta\left[u_m^{ij}(t+1) + u_c^{ij}(t+1) W_{t+1} h_{t+1}^{ij} l_{t+1}^{ij} + \left(1 - \delta_m\right)\frac{u_c^{ij}(t+1)}{g_e^{ij}(t+1)}\right] = 0 \tag{9}$$

$$v_z^{ij}(t) - u_c^{ij}(t) W_t m_t^{ij} h_t^{ij} = 0 \tag{10}$$

$$-\frac{\zeta^i v_z^{ij}(t)}{B_s^{ij}(t)} + \beta^j \Theta\left[u_c^{ij}(t+1) W_{t+1} m_{t+1}^{ij} l_{t+1}^{ij} + \left(1 - \delta_h\right)\frac{\zeta^i v_z^{ij}(t+1)}{B_s^{ij}(t+1)}\right] = 0 \tag{11}$$

A ij-type agent's optimal behavior and conditions concerning the trade-off between consumption, time and capital stock allocations are represented by equations (7) - (11), which together with equations (3), (4) and (5), characterize the equilibrium in the decentralized market economy. Equation (7) is interpretable as the first order condition for x_t^{ij}, in which we have recognized that the shadow price associated with health capital is equal to $(u_c^{ij}(t)/g_e^{ij}(t))$ at the equilibrium, whereas equation (7). Equation (10) is the condition for the optimal choice between schooling and hours of work. Similarly, equations (8), (9) and (11) refer to the optimal choices of k_{t+1}^{ij}, m_{t+1}^{ij} and h_{t+1}^{ij}, respectively. Notice that the conditions concerning the optimal choice of health and human capital take into account the effect of these choices on the these capital stocks accumulation, as well as their effects on an agent's earnings (and the direct effect of health status on agent's utility).

A representative firm produces a single good (Y_t) with capital $K_t = \sum_{i,j} \gamma^{ij} k_t^{ij}$, where γ^{ij} is the share of ij-type in the population, $\sum_{i,j} \gamma^{ij} = 1$, and the quality-adjusted labor input, $A_t = \sum_{i,j} \gamma^{ij} A_t^{ij} = \sum_{i,j} \gamma^{ij} m_t^{ij} h_t^{ij} l_t^{ij}$, which takes into account the worker's health and human capital, i.e., $Y_t = F(K_t, A_t)$. The firm operates under perfect competition and maximize profits. Factors of production are paid their marginal products, implying that $(\partial F(K_t, A_t)/\partial K_t) = R_t$ and $(\partial f(K_t, A_t)/\partial A_t) = W_t$.

The economy resource constraint for period t

$$F(K_t, A_t) + K_{t+1} = \sum_{i,j} \gamma^{ij} \left(c_t^{ij} + x_t^{ij} + e_t^{ij} + k_t^{ij} \right) \tag{12}$$

3 First-Best Optimal Policies

We assume that the planner is paternalistic utilitarian and its objective consists of the sum of utilities where $\beta^j = 1$, following for instance, O'Donoghue and Rabin (2003) and Cremer et al. (2012), among others. The reason for the difference between the planner's and the individuals' preferences resides in the (unrecognized) mistakes made by individuals. Time-inconsistent individuals underestimate the real/correct shadow prices of physical, human and health capital, as well as the shadow price of their labor. The planner's problem in Lagrangian form is as follows

$$\begin{aligned} \mathscr{L}_P^{1st} &= \sum_{t=0}^{\infty} \Theta^t \Bigg\{ \sum_{i,j} \gamma^{ij} \left[u\left(c_t^{ij}, x_t^{ij}, m_t^{ij}\right) + v\left(z_t^{ij}\right) \right] \\ &+ \eta_t \left[F(K_t, A_t) + K_{t+1} - \left(\sum_{i,j} \gamma^{ij} \left(c_t^{ij} + x_t^{ij} + e_t^{ij} + k_t^{ij} \right) \right) \right] \\ &+ \widehat{\eta}_t^{ij} \sum_{i,j} \gamma^{ij} \left[h_{t+1}^{ij} - (1-\delta_h) h_t^{ij} - B\left(s_t^{ij}\right) \right] \\ &+ \widetilde{\eta}_t^{ij} \sum_{i,j} \gamma^{ij} \left[m_{t+1}^{ij} - (1-\delta_m) m_t^{ij} - g\left(x_t^{ij}, e_t^{ij}\right) \right] \Bigg\} \end{aligned} \tag{13}$$

In the first-best, the planner's goal is to design policies that induce/help individuals to internalize the external effects of their time-inconsistent preference for immediate gratification and (in conjunction to) their cognitive ability to study. These policies are to be announced in each period and they must be part of a "surprise policy" introduced in each period, to be received in the next period, since agents do not expect to be time-inconsistent in the future. There are several ways to implement the social optimum in the decentralized economy and, in this paper, we consider two cases to emphasize the potential interactions of decisions regarding health and human capital accumulation and time and consumption allocations. Denote the socially optimal resource allocation, i.e., the solution of the planner's problem, as $\{c_t^{ij*}, x_t^{ij*}, e_t^{ij*}, s_t^{ij*}, l_t^{ij*}, k_{t+1}^{ij*}, m_{t+1}^{ij*}, h_{t+1}^{ij*}\}$ for all agents type ij and period t, and define $u^{ij*}(t) = u\left(c_t^{ij*}, x_t^{ij*}, m_t^{ij*}\right)$, $v^{ij*}(t) = v\left(z_t^{ij*}\right)$, $B^{ij*}(t) = B\left(s_t^{ij*}\right)$, $g^{ij}(t) = g\left(x_t^{ij*}, e_t^{ij*}\right)$, and $F^*(t) = F\left(K_t^*, A_t^*\right)$.

Subsidies to Wealth, Human and Health Capital Stocks. Suppose that the planner were to announce subsidies proportional to the agent's private wealth and his stocks of health and human capital to implement the social optimum in the decentralized economy. In the first-best equilibrium the planner can identify each agent ij-type and design individual specific policies accordingly. Note however that these policies must be part of a "surprise policy" introduced in each period, since the agents do not expect to be time-inconsistent in the future. Consider, for instance, the decisions made by the ij-type agent in period t, i.e. when the consumer chooses $\{c_t^{ij}, x_t^{ij}, e_t^{ij}, s_t^{ij}, l_t^{ij}, k_{t+1}^{ij}, m_{t+1}^{ij}, h_{t+1}^{ij}\}$ conditional on $\{k_t^{ij}, m_t^{ij}, h_t^{ij}\}$. Introducing subsidies at the rates S_{t+1}^{ij*}, M_{t+1}^{ij*} and H_{t+1}^{ij*} implies that the ij-type agent's budget constraint, equation (5), for $t+1$, changes to read

$$\begin{aligned} c_{t+1}^{ij} + x_{t+1}^{ij} + e^i{}_{t+1} + k_{t+2}^{ij} &= (1 + R_{t+1} - \delta_k)\left(1 + S_{t+1}^{ij}\right) k_{t+1}^{ij} + H_{t+1}^{ij} h_{t+1}^{ij} \\ &+ M_{t+1}^{ij} m_{t+1}^{ij} + W_{t+1}\left(m_{t+1}^{ij} h_{t+1}^{ij}\right) l_{t+1}^{ij} + T_{t+1}^{ij} \end{aligned}$$

in which case S_{t+1}^{ij*}, M_{t+1}^{ij*} and H_{t+1}^{ij*} directly affect the agent's choice set in period t. The lump-sum tax T_{t+1}^{ij} is such that the government's budget constraint, $(1 + R_{t+1} - \delta_k) S_{t+1}^{ij} k_{t+1}^{ij} + H_{t+1}^{ij} h_{t+1}^{ij} + M_{t+1}^{ij} m_{t+1}^{ij} = T_{t+1}^{ij}$, is satisfied for all ij-type agent and for all $t > 0$.

Proposition 1. *Suppose the government announces, in each period t, a surprise set of policies that contains subsidies proportional to the agent's private wealth and his stocks of health and human capital, and to be implemented in period $t+1$, i.e., $(1 + R_{t+1} - \delta_k)\left(1 + S_{t+1}^{ij*}\right) k_{t+1}^{ij}$, $M_{t+1}^{ij*} m_{t+1}^{ij}$ and $H_{t+1}^{ij*} h_{t+1}^{ij}$. Then the equilibrium in the decentralized economy is equivalent to the social optimum if*

$$S_{t+1}^{ij*} = \frac{1-\beta^j}{\beta^j} \tag{14}$$

$$H_{t+1}^{ij*} = \left(\frac{1-\beta^j}{\beta^j}\right)\left(\frac{1}{u_c^{ij*}(t+1)}\right)\begin{bmatrix} F_A^*(t+1) m_{t+1}^{ij} l_{t+1}^{ij} u_c^{ij*}(t+1) \\ + (1-\delta_h)\,\zeta^i \frac{v_z^{ij*}(t+1)}{B_s^{ij*}(t+1)} \end{bmatrix} \tag{15}$$

$$M_{t+1}^{ij*} = \left(\frac{1-\beta^j}{\beta^j}\right)\left(\frac{1}{u_c^{ij*}(t+1)}\right)\begin{bmatrix} u_m^{ij*}(t+1) \\ + F_A^*(t+1) h_{t+1}^{ij} l_{t+1}^{ij} u_c^{ij*}(t+1) \\ + (1-\delta_m) \frac{u_c^{ij*}(t+1)}{g_e^{ij*}(t+1)} \end{bmatrix} \tag{16}$$

Proof. See Appendix. □

Each formula in the Proposition 1 serves the purpose of eliminating a divergence between an Euler equation associated with the private optimization problem and the corresponding equation resulting from the social optimization problem. Individuals underestimate the shadow prices of physical, health and human capital and the policies aim to correct precisely that. These policies are in fact subsidies and they entail direct and indirect effects on individuals decision. The health capital subsidy H_{t+1}^{ij*} increases future welfare consumption directly through a larger output (first term in the bracket). Indirectly, it also stimulates accumulation on human capital via changes in shadow prices of leisure *vis-à-vis* education slaking that constraint (second term in the bracket). On the other hand, the policy M_{t+1}^{ij*} has two direct effects namely (i) marginal increases in the utility of health (first term in the bracket) and (ii) an expansion in aggregate output due to an increase on individuals' health status and its impact on future consumption (second term in the brackets). This policy also affects the future set of consumer's choices, i.e., the marginal increase in consumption discounted by its depreciation versus the reduction in her private health expenditures. This welfare gain is summarized by the shadow price of health capital, which is equal to $((1-\delta_m)\, u_c^{ij*}(t+1)/g_e^{ij}(t+1)) > 0$ at the equilibrium. An interpretation is that the increase in the stock of health capital leads the consumer to reduce his/her private health expenditures, *ceteris paribus*, which increases the resources available for private consumption. the agent's decision regarding future consumption *vis-à-vis* medical expenditures, changing the correspondent shadow prices (third term). Note that individuals with larger cognitive skills (low ζ) and higher non-cognitive skills (high β), *ceteris paribus*, should face lower subsidy in human and health capital subsidy, which reflects redistributive flavors of the policy package.

Subsidies to Wealth and Earnings. In our economy an individual accumulates not only physical capital but also human and health capital. An agent's earnings are, hence, determined by the health-quality of his human capital, i.e., the combination of his health and human capital. The planner can also implement the social optimum through subsidies proportional to the agent's private wealth and earnings, taking into account the interaction of the agent's time-inconsistent preference for immediate gratification (present-bias) and his cognitive ability to study.

When a single policy is in place, the planner takes into account the effect of both health and education on an agent's labor earnings. We have four terms. The first one captures the impact on future consumption due to increase in the productivity $(u_c^{ij*}(t+1)F_A^*(t+1)l_{t+1}^{ij})$, the second term captures a direct impact on health on health status (u_A^i). The third term relaxes the shadow price between future consumption (discounted by health depreciation) and medical expenditures $\big((1-\delta_m)\, u_c^{ij*}(t+1)/g_e^{ij}(t+1)h_{t+1}^{ij}\big)$, weighted by the individual's education level. The fourth term shows that it also affects the shadow price between leisure and human capital investment $\big((1-\delta_h)\, \zeta^i v_z^*(t+1)/B_s^i(t+1)\big)$, weighted by the individual's health level. Note that this policy is only slightly different from the combination of equations (15) and (16). The direct effect on productivity and utility contains marginal effects of both health and education changes. The difference is now that both effects on shadow prices - health versus future consumption/ education and leisure- are weighted by each opposed input (health and human capital) in the production function respectively. However, both input effects (health and education) have on present-bias consumption and leisure decisions are positive and this increases the optimal subsidy. Similarly, one can observe from this earning subsidy, that the larger the ζ^i (lower cognitive skill), or the lesser the non-cognitive skill (β), the larger the subsidy on health and human capital. This aims at redistribution purposes. Last, first-best policies do not require any intervention on the part of the government in the case that there is not non-cognitive issue. These results are summarized in the following proposition.

Proposition 2. *Suppose the government announces, in each period t, surprise subsidies proportional to the agent's private wealth and his earnings, to be implemented in period $t+1$,, i.e., $(1+R_{t+1}-\delta_k)\left(1+S_{t+1}^{ij*}\right)k_{t+1}^{ij}$ and $\left(1+O_{t+1}^{ij*}\right)W_{t+1}Q_{t+1}^{ij}l_{t+1}^{ij}$, where $Q_{t+1}^{ij}=m_{t+1}^{ij}h_{t+1}^{ij}$ is the health-quality ij-type human capital. With subsidies S_{t+1}^{ij*}, given by (14), and*

$$O_{t+1}^{ij*} = \left(\frac{1-\beta^j}{\beta^j}\right)\left(\frac{1}{u_c^{ij*}(t+1)F_A^*(t+1)l_{t+1}^{ij}}\right)\left[\begin{array}{l} u_A^{ij*}(t+1) \\ +F_A^*(t+1)l_{t+1}^{ij}u_c^{ij*}(t+1) \\ +(1-\delta_m)\frac{u_c^{ij*}(t+1)}{g_e^{ij}(t+1)h_{t+1}^{ij}} \\ +(1-\delta_h)\zeta^i\frac{v_z^{ij*}(t+1)}{B_s^{ij*}(t+1)m_{t+1}^{ij}} \end{array}\right] \tag{17}$$

the equilibrium in the decentralized economy is equivalent to the social optimum.

Proof. See Appendix. □

Constrained First-Best. In our first-best analysis so far, the planner could identify each ij-type and design type specific policies to recover the first-best equilibrium. Facing, however, the impossibility of knowing each agent's cognitive and non-cognitive abilities, the planner is constrained to use a single policy package for all agents. The planner's policy choice is constrained by human and health capital laws of motion and the aggregate resource constraint, equations (3), (4), and (12), respectively and non-type specific policies. Evidently, in this constrained first-best setup, the resulting optimal equilibrium is clearly sub-optimal when compared to the first-best equilibrium.

We first assume that the planner announces subsidies proportional to the agent's private wealth and his stocks of health and human capital. Since the planner cannot identify each ij-type agent and design individual specific policies accordingly, we rewrite the ij-type agent's budget constraint, equation (5), for $t \geq 1$, as follows

$$\begin{aligned} c_t^{ij} + x_t^{ij} + e_t^{ij} + k_{t+1}^{ij} &= (1 + R_t - \delta_k)\left(1 + \widehat{S}_t^*\right) k_t^{ij} + \widehat{H}_t^* h_t^{ij} \\ &+ \widehat{M}_t^* m_t^{ij} + W_t \left(m_t^{ij} h_t^{ij}\right) l_t^{ij} + \widehat{T}_t^* \end{aligned} \tag{18}$$

where $\widehat{S}_t^*$, $\widehat{M}_t^*$, and $\widehat{H}_t^*$ are subsidies proportional to the agent's private wealth and his stocks of health and human capital, respectively. The first-order conditions of this problem are straightforward and similar to problem (6). Combining the equilibrium equations of all ij-types with the planner's equilibrium conditions (solution of problem (13)), we obtain a single optimal policy package for all ij-type agents. Now, as we have only one policy for all agents, the main additional ingredient in the solutions below regards the weighted average of all individuals' allocations $\left(\sum_{i,j} \gamma^{ij}\right)$. Equation (19) has similar interpretation as Equation (14). To see this, note that if one observes individuals with the same non-cognitive skill (β), these equations are identical. Similarly, an equivalent exercise can be conducted using equations (20) and (21) using identical $\beta's$. However note that the optimal educational calls for a correction between (a weighted average) of (i) marginal effects on production, $\sum_{i,j} \gamma^{ij} F_A^*(t) m_t^{ij} l_t^{ij} u_c^{ij*}(t) - \sum_{i,j} \gamma^{ij} \beta^j F_A^*(t) m_t^{ij} l_t^{ij} u_c^{ij*}(t)$, and (ii) the marginal rate of substitution between leisure and hours of study, $\sum_{i,j} \gamma^{ij} \left(\zeta^i v_z^{ij*}(t)/B_s^{ij*}(t)\right) - \sum_{i,j} \gamma^{ij} \beta^j \left(\zeta^i v_z^{ij*}(t)/B_s^{ij*}(t)\right)$. This last term shows that even with same non-cognitive skills, this policy is different from the first best as it takes into consideration weighted average of heterogeneous cognitive skills. For the optimal health subsidy, one must add the the direct effect of health on welfare, $\sum_{i,j} \gamma^{ij} u_m^{ij*}(t) - \sum_{i,j} \gamma^{ij} \beta^j u_m^{ij*}(t)$, due to different evaluation between consumers and government, other than those that should correct for the difference in future production effects, $\sum_{i,j} \gamma^{ij} F_A^*(t) h_t^{ij} l_t^{ij} u_c^{ij*}(t) - \sum_{i,j} \gamma^{ij} \beta^j F_A^*(t) h_t^{ij} l_t^{ij} u_c^{ij*}(t)$, and on marginal rate of substitution between consumption and health, $\sum_{i,j} \gamma^{ij} \left(u_c^{ij*}(t)/g_e^{ij*}(t)\right) - \sum_{i,j} \gamma^{ij} \beta^j \left(u_c^{ij*}(t)/g_e^{ij*}(t)\right)$. These optimal policies can now give different weight to allocation of those with heterogeneous skills. The following proposition summarizes our results.

Proposition 3. *Suppose the government announces at time* 0 *a set of policies that contains subsidies proportional to the agent's private wealth and his stocks of health and human capital to be implemented in each period* $t \geq 1$, *i.e.*, $(1 + R_t - \delta_k)\left(1 + \widehat{S}_t^*\right) k_t^{ij}$, $\widehat{M}_t^* m_t^{ij}$ *and* $\widehat{H}_t^* h_t^{ij}$. *Then the* *equilibrium in the decentralized economy is equivalent to the social optimum if*

$$\widehat{S}_t^* = \frac{\sum_{i,j}\gamma^{ij}\frac{u_c^{ij*}(t-1)}{\beta^j u_c^{ij*}(t)} - \sum_{i,j}\gamma^{ij}\frac{u_c^{ij*}(t-1)}{u_c^{ij*}(t)}}{\sum_{i,j}\gamma^{ij}\frac{u_c^{ij*}(t-1)}{u_c^{ij*}(t)}} \tag{19}$$

$$\widehat{H}_t^* = \left(\frac{1}{\sum_{i,j}\gamma^{ij}\beta^j u_c^{ij*}(t)}\right)\left\{\begin{array}{l}\sum_{i,j}\gamma^{ij}F_A^*(t)m_t^{ij}l_t^{ij}u_c^{ij*}(t) - \sum_{i,j}\gamma^{ij}\beta^j F_A^*(t)m_t^{ij}l_t^{ij}u_c^{ij*}(t)\\ +(1-\delta_h)\left[\sum_{i,j}\gamma^{ij}\frac{\zeta^i v_z^{ij*}(t)}{B_s^{ij*}(t)} - \sum_{i,j}\gamma^{ij}\beta^j\frac{\zeta^i v_z^{ij*}(t)}{B_s^{ij*}(t)}\right]\end{array}\right\} \tag{20}$$

$$\widehat{M}_t^* = \left(\frac{1}{\sum_{i,j}\gamma^{ij}\beta^j u_c^{ij*}(t)}\right)\left\{\begin{array}{l}\sum_{i,j}\gamma^{ij}u_m^{ij*}(t) - \sum_{i,j}\gamma^{ij}\beta^j u_m^{ij*}(t)\\ +\sum_{i,j}\gamma^{ij}F_A^*(t)h_t^{ij}l_t^{ij}u_c^{ij*}(t) - \sum_{i,j}\gamma^{ij}\beta^j F_A^*(t)h_t^{ij}l_t^{ij}u_c^{ij*}(t)\\ +(1-\delta_m)\left[\sum_{i,j}\gamma^{ij}\frac{u_c^{ij*}(t)}{g_e^{ij*}(t)} - \sum_{i,j}\gamma^{ij}\beta^j\frac{u_c^{ij*}(t)}{g_e^{ij*}(t)}\right]\end{array}\right\} \tag{21}$$

Proof. See Appendix. □

Next, we assume that the planner can commit to policies that subsidies the agent's private wealth and earnings, taking into account the interaction of the agent's time-inconsistent preference for immediate gratification (present-bias) and his cognitive ability to study. Similar to problem (6), in this case, a ij-type agent chooses allocations to maximize the utility function (2) subject to the laws of motion, equations (3) and (4), taking the initial physical, health and human capital stocks, k_0^{ij}, m_0^{ij} and h_0^{ij}, as exogenously given and the modified budget constraint as follows:

$$c_t^{ij} + x_t^{ij} + e_t^{ij} + k_{t+1}^{ij} = \left(1+\widehat{O}_t^*\right)W_t\left(m_t^{ij}h_t^{ij}\right)l_t^{ij} + (1+R_t-\delta_k)\left(1+\widehat{S}_t^*\right)k_t^{ij} + \widehat{T}_t^* \tag{22}$$

The interpretation of equation (23), Proposition (4) is similar to (17), Proposition (2). When all individuals have the same $\beta' s$, all terms become correspondent between the two expression with the only difference the this new policy is a weighted average of all individuals allocations. The first term must correct for the direct effect of health on welfare, $\sum_{i,j}\gamma^{ij}\left(u_m^{ij*}(t)/h_t^{ij}\right) - \sum_{i,j}\gamma^{ij}\beta^j\left(u_m^{ij*}(t)/h_t^{ij}\right)$; the second term addresses the marginal effect on the production side due changes in marginal change of effective labor supply, $\sum_{i,j}\gamma^{ij}F_A^*(t)l_t^{ij}u_c^{ij*}(t) - \sum_{i,j}\gamma^{ij}\beta^j F_A^*(t)l_t^{ij}u_c^{ij*}(t)$. The third and fourth term aims to slack the shadow prices between future consumption and health and leisure and future benefit of education, $(1-\delta_m)\sum_{i,j}\gamma^{ij}\left(u_c^{ij*}(t)/g_e^{ij}(t)h_t^{ij}\right) - \sum_{i,j}\gamma^{ij}\beta^j\left(u_c^{ij*}(t)/g_e^{ij}(t)h_t^{ij}\right)$ and $(1-\delta_h)\sum_{i,j}\gamma^{ij}\left(\zeta^i v_z^{ij*}(t)/B_s^{ij*}(t)m_t^{ij}\right) - \sum_{i,j}\gamma^{ij}\beta^j\left(\zeta^i v_z^{ij*}(t)/B_s^{ij*}(t)m_t^{ij}\right)$, respectively. The following proposition summarizes our results.

Proposition 4. *Suppose the government announces, in each period t, surprise subsidies proportional to the agent's private wealth and his earnings, to be implemented in period $t+1$,, i.e.,* $(1+R_t-\delta_k)\left(1+\widehat{S}_t^*\right)k_t^{ij}$ *and* $\left(1+\widehat{O}_t^*\right)W_t Q_t^{ij}l_t^{ij}$, *where* $Q_t^{ij} = m_t^{ij}h_t^{ij}$ *is the health-quality ij-type* *human capital. With subsidies $\widehat{S}_t^*$, given by (19), and*

$$\widehat{O}_t^* = \left(\frac{1}{\sum_{i,j}\gamma^{ij}\beta^j u_c^{ij*}(t)F_A^*(t)l_t^{ij}}\right)\left\{\begin{array}{l} \sum_{i,j}\gamma^{ij}\frac{u_m^{ij*}(t)}{h_t^{ij}} - \sum_{i,j}\gamma^{ij}\beta^j\frac{u_m^{ij*}(t)}{h_t^{ij}} \\ +\sum_{i,j}\gamma^{ij}F_A^*(t)l_t^{ij}u_c^{ij*}(t) - \sum_{i,j}\gamma^{ij}\beta^j F_A^*(t)l_t^{ij}u_c^{ij*}(t) \\ +(1-\delta_m)\left[\sum_{i,j}\gamma^{ij}\frac{u_c^{ij*}(t)}{g_e^{ij}(t)h_t^{ij}} - \sum_{i,j}\gamma^{ij}\beta^j\frac{u_c^{ij*}(t)}{g_e^{ij}(t)h_t^{ij}}\right] \\ +(1-\delta_h)\left[\sum_{i,j}\gamma^{ij}\frac{\zeta^i v_z^{ij*}(t)}{B_s^{ij*}(t)m_t^{ij}} - \sum_{i,j}\gamma^{ij}\beta^j\frac{\zeta^i v_z^{ij*}(t)}{B_s^{ij*}(t)m_t^{ij}}\right] \end{array}\right\} \quad (23)$$

Proof. See Appendix. □

4 Second-Best Optimal Policies

At the beginning of each period, the government announces its policy package and individuals behave competitively. In the second-best setting, when designing an optimal policy, the government takes into account the equilibrium reactions by private agents to the tax system. We assume that the government does not set taxes sequentially but commits to a policy package at time 0. That is, it is assumed that the government can commit itself to the policies that will be in place arbitrarily far into the future.[3]

Subsidies to Wealth, Human and Health Capital Stocks. The planner's policy choice is constrained by ij-type agents first-order order conditions, equivalent to equations (7) - (11) when policies $\widetilde{S}_t^*$, $\widetilde{H}_t^*$, and $\widetilde{M}_t^*$ are considered, their budget constraints, equation (18), the government budget constraint, $(1 + R_t - \delta_k)\,\widetilde{S}_t^* k_t^{ij} + \widetilde{H}_t^* h_t^{ij} + m_t^{ij}\widetilde{M}_t^* = \widetilde{T}_t^*$, $\forall t > 1$, and human and health capital laws of motion, equations (3) and (4), respectively. To improve the paper's readability, the planner's second-best problem in Lagrangian form is presented in the Appendix.

Proposition 5. *Suppose the government announces at time* 0 *a set of policies that contains subsidies proportional to the agent's private wealth and his stocks of health and human capital to be implemented at time* $t = 1$, *i.e.,* $(1 + R_1 - \delta_k)\,\widetilde{S}_1^* k_1^{ij}$, $m_1^{ij}\widetilde{M}_1^*$ *and* $\widetilde{H}_1^* h_1^{ij}$. *Then the policy package*

[3] We follow a large literature in assuming that the government can commit to follow a long-term program for taxing physical capital, human capital and health capital. We assume that there are institutions that effectively solve the time inconsistency problem so that the government can commit to the tax plan it announces in the initial period (Ramsey (1927), Judd (1985), Chamley (1986), and Chari et al. (1994, 1995))

is as follows:

$$\widetilde{S}_1^* = \frac{\sum_{i,j}\gamma^{ij}\lambda_0^{ij}\left(\sum_{i,j}\gamma^{ij}u_c^{ij*}(1)-\sum_{i,j}\gamma^{ij}\beta^j u_c^{ij*}(1)\right)}{\sum_{i,j}\gamma^{ij}\lambda_0^{ij}\left(\sum_{i,j}\gamma^{ij}u_c^{ij*}(1)-\sum_{i,j}\gamma^{ij}\beta^j u_c^{ij*}(1)\right)-\eta_t\left(\sum_{i,j}\gamma^{ij}u_c^{ij*}(1)\right)} \tag{24}$$

$$\widetilde{H}_1^* = \Psi(1)\left\{\begin{array}{l} F_A^*(1)\left[\sum_{i,j}\gamma^{ij}\left(\widetilde{\psi}_1^{ij}u_c^{ij*}(1)m_1^{ij}-\lambda_1^{ij}m_1^{ij}l_1^{ij}\right)\Upsilon(0)\right. \\ \left.-\left(\sum_{i,j}\gamma^{ij}\beta^j u_c^{ij*}(1)m_1^{ij}l_1^{ij}\right)\Omega(1)\right] \\ +(1-\delta_h)\left[\left(\sum_{i,j}\gamma^{ij}\widehat{\eta}_0^{ij}\right)\Upsilon(0)-\left(\sum_{i,j}\gamma^{ij}\left(\lambda_1^{ij}-\eta_1\right)\right)\Upsilon(0)\right] \end{array}\right\} \tag{25}$$

$$\widetilde{M}_1^* = ? \tag{26}$$

where $\Psi(1)=\left\{\left(\lambda_1^{ij}-\eta_1\right)\Upsilon(0)+\Omega(1)\right\}^{-1}$, $\Omega(1)=\sum_{i,j}\gamma^{ij}\left(\widehat{\eta}_0^{ij}+\eta_0 u_c^{ij*}(1)l_1^{ij}F_A^*(1)\right)$, *and* $\Upsilon(0)=\sum_{i,j}\gamma^{ij}\left(\frac{\zeta^i v_z^{ij*}(0)}{B_s^{ij*}(0)m_0^{ij}}\right)$.

Proof. See Appendix. □

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Proposition 6. *Suppose the government announces at time* 0 *a set of policies that contains subsidies proportional to the agent's private wealth and his stocks of health and human capital to be implemented in each period* $t>1$, *i.e.,* $(1+R_t-\delta_k)\,\widetilde{S}_t^* k_t^{ij}$, $m_t^{ij}\widetilde{M}_t^*$ *and* $\widetilde{H}_t^* h_t^{ij}$. *If the solution to the planner's problem converges to a steady state, then in a steady state, the policy package,* $\forall t>1$, *is as follows:*

$$\widetilde{S}_t^* = 0; \tag{27}$$

$$\widetilde{H}_t^* = \Psi(t)\left\{\begin{array}{l} F_A^*\left[\sum_{i,j}\gamma^{ij}\left(\widetilde{\psi}^{ij}u_c^{ij*}m^{ij}-\lambda^{ij}m^{ij}l^{ij}\right)\Upsilon\right. \\ \left.-\left(\sum_{i,j}\gamma^{ij}u_c^{ij*}m^{ij}l^{ij}\right)\Omega\right] \\ +(1-\delta_h)\left[\left(\sum_{i,j}\gamma^{ij}\widehat{\eta}^{ij}\right)\Upsilon-\left(\sum_{i,j}\gamma^{ij}\left(\lambda^{ij}-\eta\right)\right)\Upsilon\right] \end{array}\right\} \tag{28}$$

$$\widetilde{M}_t^* = ?\{\} \tag{29}$$

where $\Psi=\{(\lambda^{ij}-\eta)\,\Upsilon+\Omega\}^{-1}$, $\Omega=\sum_{i,j}\gamma^{ij}\left(\widehat{\eta}^{ij}+\eta u_c^{ij*}l^{ij}F_A^*\right)$, *and* $\Upsilon=\sum_{i,j}\gamma^{ij}\left(\frac{\zeta^i v_z^{ij*}}{B_s^{ij*}m^{ij}}\right)$.

Proof. See Appendix. □

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Subsidies to Wealth and Earnings.

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Proposition 7. *Suppose the government announces, in each period t, surprise subsidies proportional to the agent's private wealth and his earnings, to be implemented at time $t = 1$, i.e., $(1 + R_1 - \delta_k)\left(1 + \widetilde{S}_1^*\right) k_1^{ij}$ and $\left(1 + \widetilde{O}_1^*\right) W_1 Q_1^{ij} l_1^{ij}$, where $Q_1^{ij} = m_1^{ij} h_1^{ij}$ is the health-quality ij-type human capital. Then the policy package includes subsidy $\widetilde{S}_1^*$, similar to the expression in Proposition (5), and*

$$\widetilde{O}_1^* = \widetilde{\Psi}(1) \left\{ \begin{array}{l} \left(\sum_{i,j} \gamma^{ij} \beta^j \frac{u_m^{ij*}(1)}{h_1^{ij}}\right) \widetilde{\Omega}(1) - \left(\sum_{i,j} \gamma^{ij} \frac{u_m^{ij*}(1)}{h_1^{ij}}\right) \widetilde{\Upsilon}(1) \\ +F_A^*(1) \left[\left(\sum_{i,j} \gamma^{ij} \beta^j u_c^{ij*}(1) l_1^{ij}\right) \widetilde{\Omega}(1) - \left(\sum_{i,j} \gamma^{ij} \lambda_1^{ij} l_1^{ij} - \sum_{i,j} \gamma^{ij} \widetilde{\psi}_1^{ij} l_1^{ij}\right) \widetilde{\Upsilon}(1)\right] \\ +(1-\delta_m) \left[\left(\sum_{i,j} \gamma^{ij} \beta^j \frac{u_c^{ij*}(1)}{g_e^{ij}(1) h_1^{ij}}\right) \widetilde{\Omega}(1) - \left(\sum_{i,j} \gamma^{ij} \frac{\widehat{\eta}_1^{ij}}{h_1^{ij}}\right) \widetilde{\Upsilon}(1)\right] \\ +(1-\delta_h) \left[\left(\sum_{i,j} \gamma^{ij} \beta^j \frac{\zeta^i v_z^{ij*}(1)}{B_s^{ij*}(1) m_1^{ij}}\right) \widetilde{\Omega}(1) - \left(\sum_{i,j} \gamma^{ij} \frac{\widetilde{\eta}_1^{ij}}{m_1^{ij}}\right) \widetilde{\Upsilon}(1)\right] \end{array} \right\}$$

where $\widetilde{\Psi}(1) = \left\{F_A^*(1)\left[\left(\sum_{i,j} \gamma^{ij}\left[\lambda_1^{ij} - \eta_1\right] l_1^{ij}\right) \widetilde{\Upsilon}(1) - \left(\sum_{i,j} \gamma^{ij} \beta^j u_c^{ij*}(1) l_1^{ij}\right) \widetilde{\Omega}(1)\right]\right\}^{-1}$, $\widetilde{\Omega}(1) = \sum_{i,j} \gamma^{ij} \left(\frac{\widehat{\eta}_1^{ij}}{h_1^{ij}} + \right.$ *and* $\widetilde{\Upsilon}(1) = \sum_{i,j} \gamma^{ij} \left(\frac{u_c^{ij*}(1)}{g_e^{ij}(1) h_1^{ij}} + \frac{\zeta^i v_z^{ij*}(1)}{B_s^{ij*}(1) m_1^{ij}}\right)$.

Proof. See Appendix. □

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Proposition 8. *Suppose the government announces, in each period t, surprise subsidies proportional to the agent's private wealth and his earnings, to be implemented in each period $t > 1$, i.e., $(1 + R_t - \delta_k)\left(1 + \widetilde{S}_t^*\right) k_t^{ij}$ and $\left(1 + \widetilde{O}_t^*\right) W_t Q_t^{ij} l_t^{ij}$, where $Q_t^{ij} = m_t^{ij} h_t^{ij}$ is the health-quality ij-type human capital. If the solution to the planner's problem converges to a steady state, then in a steady state, the policy package includes subsidies $\widetilde{S}_t^*$, similar to the expressions in Proposition (6), and*

$$\widetilde{O}^* = \widetilde{\Psi} \left\{ \begin{array}{l} \left(\sum_{i,j} \gamma^{ij} \frac{u_m^{ij*}(1)}{h_1^{ij}}\right) \left(\widetilde{\Omega} - \widetilde{\Upsilon}\right) \\ +F_A^* \left[\left(\sum_{i,j} \gamma^{ij} u_c^{ij*} l^{ij}\right) \widetilde{\Omega} - \left(\sum_{i,j} \gamma^{ij} \lambda^{ij} l^{ij} - \sum_{i,j} \gamma^{ij} \widetilde{\psi}^{ij} l^{ij}\right) \widetilde{\Upsilon}\right] \\ +(1-\delta_m) \left[\left(\sum_{i,j} \gamma^{ij} \frac{u_c^{ij*}}{g_e^{ij} h^{ij}}\right) \widetilde{\Omega} - \left(\sum_{i,j} \gamma^{ij} \frac{\widehat{\eta}^{ij}}{h^{ij}}\right) \widetilde{\Upsilon}\right] \\ +(1-\delta_h) \left[\left(\sum_{i,j} \gamma^{ij} \frac{\zeta^i v_z^{ij*}}{B_s^{ij*} m^{ij}}\right) \widetilde{\Omega} - \left(\sum_{i,j} \gamma^{ij} \frac{\widetilde{\eta}^{ij}}{m^{ij}}\right) \widetilde{\Upsilon}\right] \end{array} \right\}$$

where $\widetilde{\Psi} = \left\{F_A^*\left[\left(\sum_{i,j} \gamma^{ij}\left[\lambda^{ij} - \eta\right] l^{ij}\right) \widetilde{\Upsilon} - \left(\sum_{i,j} \gamma^{ij} u_c^{ij*} l^{ij}\right) \widetilde{\Omega}\right]\right\}^{-1}$, $\widetilde{\Omega} = \sum_{i,j} \gamma^{ij} \left(\frac{\widehat{\eta}^{ij}}{h^{ij}} + \frac{\widetilde{\eta}^{ij}}{m^{ij}}\right)$, *and* $\widetilde{\Upsilon} = \sum_{i,j} \gamma^{ij} \left(\frac{u_c^{ij*}}{g_e^{ij} h^{ij}} + \frac{\zeta^i v_z^{ij*}}{B_s^{ij*} m^{ij}}\right)$.

Proof. See Appendix. □

5 Numerical Results

5.1 Functional Forms and Model Parametrization

We now use numerical methods to simulate a numerical version of our model. We use these results to illustrate our main points and to explore further the relationship between human and health capital of present-bias agents. We assume the instantaneous utility function $u\left(c_t^{ij}, x_t^{ij}, m_t^{ij}\right) = log\left(c_t^{ij}\right) + log\left(x_t^{ij}\right) + \phi_1 log\left(m_t^{ij}\right)$ and $v\left(z_t^{ij}\right) = \phi_2 \frac{(1-\zeta^i s_t^{ij} - l_t^{ij})^{1-\eta}}{(1-\eta)}$, where the weights on health status and leisure are normalized to one, i.e., $\phi_1 = \phi_2 = 1$. The conventional utility discount factor $\Theta^t = 1/(1+\theta)^t$ with utility discount rate θ is set to $\Theta = 0.99$ which is consistent with a steady-state real interest rate of one percent (per quarter). For present purposes, we assume $\eta = 2.0$.

In this exercise we consider four types of agents who are different regarding their time-inconsistent preference for immediate gratification and their cognitive ability to study. We assume that agents have same present-bias towards consumption and leisure. Some agents discount the future more heavily and have greater present-bias towards consumption and leisure ($\beta^H = 0.85$), than others ($\beta^L = 0.90$). To an agent with high cognitive ability we assign $\zeta^H = 0.5$, i.e. she can accomplish more for each unit of time dedicated to study and, hence experiences a lower leisure cost of studying. We set $\zeta^L = 0.8$ to a low cognitive ability individual.

Production technology is a constant returns Cobb-Douglas specification of the form $F(K_t, A_t) = K_t^{\alpha} A_t^{1-\alpha}$, where $K_t = \sum_{i,j} k_t^{ij}$ and $A_t = \sum_{i,j} A_t^{ij} = \sum_{i,j} m_t^{ij} h_t^{ij} l_t^{ij}$ and we set $\alpha = 0.33$. The health production and human capital accumulation functions are, respectively, $g\left(x_t^{ij}, e_t^{ij}\right) = D_1\left(e_t^{ij}\right)^{\gamma} - D_2 x_t^{ij}$ and $B\left(s_t^{ij}\right) = B_1\left(s_t^{ij}\right)^{\theta}$, and we set $D_1 = D_2 = 0.25$, $\gamma = 0.50$, $B_1 = 0.25$, and $\theta = 0.85$. We assume that physical capital does not depreciates and the depreciation rates of health stock and human capital are $\delta_h = \delta_m = 0.10$.

5.2 First-best allocations and policies

Homogeneous cognitive and non-cognitive skills agents. In this section we present the results for our benchmark parameterization when the economy is populated by a single type of agent. That is, all agents are identical regarding their cognitive ability as well as their present-bias discounting (Table I). Compared to the planner's optimal allocations, agents' allocations are lower due to present-bias discount, except for hours worked. The time-inconsistent preference for immediate gratification leads the agent to work more than what would be optimal, for any cognitive ability ζ. Agents adjust their leisure by working more and studying less than the (planner's first-best) optimal. From our numerical exercise, we can also compute a measure of the value of the interaction cognitive and non-cognitive abilities, in terms of welfare. It is clear that while better cognitive skills are welfare enhancing, time-inconsistent preferences decrease agents' welfare. Among the economies we study in Table I the higher welfare comes from the fact that agents are enjoying more consumption and a better health status. Steady state welfare is higher in an economy with less time-inconsistent agents and high cognitive skills.

Comparing agents with same discounting but different cognitive abilities (Economy I vs. Economy II; Economy III vs. Economy IV), the difference between their allocations are driven by their ability to sacrifice less leisure to accumulate human capital. Although agents work the same number of hours and experience same leisure time, agents with high cognitive ability consume more, enjoy a better health status and accumulate more physical and human capital. On the other hand,

Table I: Homogeneous agents: first-best allocations and policies

		Economy I	Economy II	Economy III	Economy IV
		$\beta^L = 0.90$	$\beta^L = 0.90$	$\beta^H = 0.85$	$\beta^H = 0.85$
		$\zeta^L = 0.80$	$\zeta^H = 0.50$	$\zeta^L = 0.80$	$\zeta^H = 0.50$
Allocations	c^{ij*}	0.06	0.13	0.03	0.05
	x^{ij*}	0.04	0.08	0.02	0.04
	e^{ij*}	0.05	0.10	0.01	0.03
	l^{ij*}	0.38	0.37	0.39	0.39
	s^{ij*}	0.18	0.29	0.14	0.23
	m^{ij*}	0.43	0.60	0.24	0.34
	k^{ij*}	0.42	0.85	0.10	0.21
	h^{ij*}	0.58	0.86	0.48	0.71
Policies	S^{ij*}	0.11	0.11	0.18	0.18
	H^{ij*}	4.93	7.19	7.83	11.41
	M^{ij*}	1.81	2.70	2.88	4.29
	T^{ij*}	36.98	80.15	58.73	127.30
	O^{ij*}	2.48	2.46	3.94	3.90
	T^{ij*}	36.64	79.11	58.20	125.64
Welfare	U^{ij*}	-8.81	-7.12	-11.02	-9.28
	U^{P*}	-1.00	0.63	-1.00	0.63

Note: Allocations of ij-type at time t, before optimal policies of ij-type are announced in time t, to be implemented at time $t+1$.

if we compare an economy where agents have different present-bias, but the same cognitive ability (Economy I vs. Economy III and Economy II vs. Economy IV), it becomes clear the effect of the time-inconsistent preference for immediate gratification on agent's allocations. Agents with high present-bias, or alternatively less patient, tend consume less of both the ordinary and the unhealthy good, spend less in health-care. Although these agents have similar cognitive ability, those with higher present-bias study less than their counterparts with lower present-bias, and consequently accumulate less human capital. Similar pattern can be identified with respect to physical capital accumulation and health status.

To implement the social optimum in the decentralized economy, suppose that the planner were to announce, in each period, that agents will receive a set of policies in the next period, which are related to individuals' private wealth and human capital and health stocks. These subsidies are financed by a lump-sum tax and must be part of a 'surprise policy' introduced in each period, since the consumer does not expect to be time-inconsistent in the future. As discussed in Section 3, there are several ways to implement the social optimum in the decentralized economy and we have considered two cases. Table I presents the numerical counterpart of Propositions 1 and 2, i.e., (i) subsidies proportional to the stock of health capital and to the stock of human capital and

(ii) a single subsidy proportional to a worker's quality of human capital. These individual specific policies are such that, once they are introduced, first-best allocations and welfare is recovered. The difference between the decentralized market economy and the social optimum arises as time-inconsistent individuals underestimate the shadow prices of (physical, human and health) capital stocks. In addition, agents of different cognitive abilities make rather different choices regarding their optimal allocations in a decentralized economy. The policy required to internalize the external time-inconsistency preference effect and its interaction with an individual cognitive ability must be designed to make the individual value physical capital, human capital and health capital in the same way as the social planner. The subsidy towards the stock of physical capital captures the present-bias aspect of this effect and it is higher when agents are more time-inconsistent.

The subsidies proportional to the stock of health capital and to the stock of human capital are higher for those agents that already have higher level of both stocks. The subsidies need to make the individual value human capital and health capital as the planner values must be bigger as they are already experiencing high levels of both when compared to otherwise similar agents (for instance, Economy I vs Economy II). Since these subsidies are financed by a lump-sum tax, these agents also pay a higher tax to balance the government budget. Interestingly, if planner uses a single subsidy proportional to a worker's quality of human capital to implement the first-best, it has to take into account the interaction not only between human and health capital on agent's earnings but also between the agent's present-bias and cognitive ability. The subsidy O^{ij*} is slightly bigger for agents with lower cognitive ability (for example, Economy I vs. Economy II), but smaller for those less time-inconsistent (Economy I vs. Economy III).

Heterogeneous cognitive and non-cognitive skills agents. Our next numerical exercise considers an economy populated by agents different cognitive and non-cognitive abilities. Agents are heterogeneous either with respect to their leisure cost of education (cognitive skill, ζ) or their time-inconsistent preference for immediate gratification (non-cognitive skill, β). Although agents are similar in one dimension, their optimal allocations might differ substantially with interesting implications for the optimal first-best policies.

We first consider the case where agents have similar cognitive abilities but differ with respect to their present-bias discounting (Table II). Agents with stronger preference for immediate gratification, i.e., β^H, do not accumulate physical capital ($k^{iH*} = 0$), which leads the agent to work much more and study less compared to his counterpart of same cognitive ability but weaker present-bias. In this case, the economy interest rate is determined by the discount rate of patient agents (β^L). This feature affects the relative prices of the three types of capital in our economy, i.e., physical, human and health capital. In fact, more patient agents experience a better health status and accumulate more human capital. The fact that less patient agents do not save leads the agent to improve his health status through spending in health care as a way to compensate the drop in human capital and the combine effect on the individual's earnings.

Table II: Heterogeneous agents: first-best allocations and policies

		Economy V		Economy VI	
		$\zeta^L = 0.8$		$\zeta^H = 0.5$	
		$\beta^L = 0.90$	$\beta^H = 0.85$	$\beta^L = 0.90$	$\beta^H = 0.85$
Allocations	c^{ij*}	0.06	0.03	0.13	0.07
	x^{ij*}	0.04	0.03	0.08	0.05
	e^{ij*}	0.05	0.03	0.10	0.06
	l^{ij*}	0.38	0.45	0.38	0.44
	s^{ij*}	0.18	0.16	0.29	0.26
	m^{ij*}	0.44	0.33	0.61	0.47
	k^{ij*}	0.39	0.00	0.80	0.00
	h^{ij*}	0.59	0.54	0.87	0.80
Policies	S^{ij*}	0.11	0.18	0.11	0.18
	H^{ij*}	4.49	10.79	6.51	15.82
	M^{ij*}	1.77	3.16	2.64	4.70
	T^{ij*}	41.35	26.27	89.67	57.44
	O^{ij*}	2.57	3.56	2.55	3.53
	T^{ij*}	41.04	25.49	88.71	55.15
Welfare	U^{ij*}	-8.84	-10.45	-7.14	-8.71
	U^{Pij*}	-0.81	-1.98	0.81	-0.33
	U^{P*}	-2.80		0.49	

Note: Allocations of ij-type at time t, before optimal policies of ij-type are announced in time t, to be implemented at time $t+1$.

We observe some redistributive role of the first-best optimal policies when there are two types of same cognitive ability but different present-bias in the economy. In Table II, notice that subsidies are higher to those agents with a stronger time-inconsistent preference for immediate gratification, i.e., $\beta^H = 0.85$ (Economy V vs. Economy III or Economy VI vs. Economy IV), while more patient agents ($\beta^L = 0.90$) receive lower subsidies (Economy V vs. Economy I or Economy VI vs. Economy II). Agents pay different lump-sum taxes, but the planner can subsidize human and health capital at different rates to reach first-best allocations. In this case, the planner offers a higher subsidy to those that underestimate the shadow prices of physical, human an heath capital more to make them value these capital stocks in the same way as the planner does. Notice that this results holds regardless the agents cognitive abilities (Economy V vs. Economy VI), however those with more cognitive abilities, i.e., $\zeta^H = 0.50$ (Economy VI) receive higher subsidies than their less cognitive skilled counterparts (Economy V).

The redistributive features of these first-best policies can also be seen when we compare the lump-sum taxes paid by agents when they are alone in the economy versus the situation when they are with others of same cognitive ability but different present-bias. Compare, for instance, Economy II and Economy VI. The HL-type agent (i.e., $\zeta^H = 0.50$, $\beta^L = 0.90$) pays higher lump-sum taxes in Economy VI (89.67) than in Economy II (80.15). The planner collects more on lump-sum taxes from the higher ability individual to be able to finance a higher subsidy to those with lower skills in the same economy. Interestingly, subsidies are of similar magnitude if they are applied to the agent's earnings (policy O^{ij*}, Economy V vs. Economy VI), being determined by the agent's cognitive ability. This is mainly due to the fact that this subsidy takes into account the combined effect of labor, human and health capital on the agent's earnings, where some agents are more mistaken about their preference for immediate gratification than others.

Table III presents our results for an economy where agents have the same present-bias discounting but different cognitive abilities. In this case, the optimal allocations of these agents are very similar to the those they would have if they were in an economy populated only by agents of the same type (Table I). The key difference between the two cases presented in the table below comes from the fact that in these economies agents have different time-inconsistent preference for immediate gratification

Table III: Heterogeneous agents: first-best allocations and policies

		Economy VII		Economy VIII	
		$\beta^L = 0.90$		$\beta^H = 0.85$	
		$\zeta^L = 0.8$	$\zeta^H = 0.5$	$\zeta^L = 0.8$	$\zeta^H = 0.5$
Allocations	c^{ij*}	0.06	0.13	0.03	0.05
	x^{ij*}	0.04	0.08	0.02	0.04
	e^{ij*}	0.05	0.10	0.01	0.03
	l^{ij*}	0.37	0.38	0.39	0.39
	s^{ij*}	0.18	0.29	0.14	0.23
	m^{ij*}	0.43	0.61	0.24	0.34
	k^{ij*}	0.44	0.83	0.10	0.21
	h^{ij*}	0.58	0.87	0.48	0.71
Policies	S^{ij*}	0.11	0.11	0.18	0.18
	H^{ij*}	3.64	5.99	7.83	11.41
	M^{ij*}	1.58	2.44	2.88	4.29
	T^{ij*}	29.07	54.46	58.73	127.30
	O^{ij*}	2.54	2.39	3.94	3.90
	T^{ij*}	28.85	53.64	58.20	125.64
Welfare	U^{ij*}	-8.79	-7.13	-11.02	-9.28
	U^{Pij*}	-1.25	0.13	-1.00	0.63
	U^{P*}	-1.12		-0.37	

Note: Allocations of ij-type at time t, before optimal policies of aij-type are announced in time t, to be implemented at time $t+1$.

When agents are less time-inconsistent (Economy VII), the presence of individuals of heterogeneous cognitive abilities in the economy allows the planner to design policies that subsidize and (lump-sum) tax both types less to reach the first-best optimum (compared to Economy I and II, Table I). Here we can also observe some redistributive role of the first-best policies. High cognitive ability agents receive higher subsidies but pay higher taxes as well. The tax burden goes down for both agents, but it drops more for the least cognitive skilled (LL-type) individual. Comparing Economy VII and Economy VIII, time-inconsistent individuals underestimate shadow prices of capital stocks and labor more and, as a consequence, the required policies to internalize the external effect of present-bias discounting include larger subsidies to physical, human and health capital. Similar feature is observed if the planner considers a subsidy to the agent's earnings (policy O^{ij*}).

Finally, we consider an economy with all four ij-types of agents, i.e., LL, LH, HL, and HH. The policy packages are designed by the planner taking into account heterogeneous agents in the economy and the individual policies to recover the first-best allocations of all four ij-types of agents. The relevant comparison here is between the individual allocations and policies when the economy is populated by homogeneous (Table I) and heterogeneous (Table IV) agents. The first noticeable difference is the physical capital accumulation of more present-bias agents, i.e., $\beta^H = 0.85$. Imposing that these agents do not save in equilibrium leads them to consume and work more, as well as to accumulate more health and human capital. On the hand, low present-bias agents ($\beta^L = 0.90$) save the same amount - in Economy IX, Table IV, physical capital accumulation is determined by the discount factor of the patient agents. Interestingly, agents' allocations do not necessarily change in the same direction. For instance, low cognitive ability agents (LL-type) consume more of the unhealthy good and spend less time on schooling, whereas the high cognitive ability agents (LH-type) consume less of such good and study more hours. These choices affect agents' health and human capital stocks - they are lower for LL-type and higher for LH-type agents.

Regarding the policy packages, subsidies and lump-sum taxes are significantly lower. In each economy studied in Table I, Economy I - IV, individuals were heavily subsidized and taxed to recover the first-best allocations. When all agents of different abilities are considered, the planner has to average out the utility of all agents when designing first-best optimal policies. For a given present-bias discounting, high cognitive ability agent receive larger subsidies but also pay more lump-sum taxes. The opposite is observed when the planner offers subsidies to the agent's earnings (O^{ij*}), which are determined by the health-quality of his human capital, i.e., the combination of his health and human capital - low cognitive ability Lj-types experience higher subsidies to their earnings relative to their high cognitive ability (Hj-types) counterparts.

Constrained First-Best: single policy and heterogeneous cognitive and non-cognitive skills agents. Recall that the constrained first-best problem is such that the planner's goal is to maximize agents' welfare subject to the economy feasibility constraint and to raising set revenues through non-type specific policies.

Table IV: Heterogeneous agents: first-best allocations and policies

		Economy IX			
		$\beta^L = 0.90$ $\zeta^L = 0.80$	$\beta^L = 0.90$ $\zeta^H = 0.50$	$\beta^H = 0.85$ $\zeta^L = 0.80$	$\beta^H = 0.85$ $\zeta^H = 0.50$
Allocations	c^{ij*}	0.06	0.13	0.03	0.07
	x^{ij*}	0.05	0.08	0.03	0.05
	e^{ij*}	0.04	0.11	0.03	0.06
	l^{ij*}	0.34	0.39	0.45	0.44
	s^{ij*}	0.16	0.30	0.16	0.26
	m^{ij*}	0.40	0.64	0.33	0.47
	k^{ij*}	0.60	0.60	0.00	0.00
	h^{ij*}	0.54	0.90	0.54	0.80
Policies	S^{ij*}	0.11	0.11	0.18	0.18
	H^{ij*}	0.08	0.15	0.12	0.16
	M^{ij*}	0.20	0.33	0.27	0.40
	T^{ij*}	0.20	0.42	0.15	0.32
	O^{ij*}	1.93	1.54	2.17	1.96
	T^{ij*}	0.23	0.45	0.19	0.35
Welfare	U^{ij*}	-8.65	-7.23	-10.45	-8.71
	U^{P*}	-0.33	-1.99	0.53	-0.26
	U^{P*}		-2.05		

Note: Allocations of ij-type at time t, before optimal policies of aij-type are announced in time t, to be implemented at time $t+1$.

5.3 Second-best allocations and policies

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6 Conclusions

The main goal of this paper is to investigate the role cognitive and non-cognitive skills on optimal taxation. We consider cognitive skills associated with schooling and human capital accumulation decisions while soft skill are primarily related to the trade-off between consumption of health and unhealthy food, health status and leisure. We assume that the instantaneous utility depends on the current consumption of healthy and unhealthy goods, current health status (stock of health capital) and leisure. An agent's stock of health capital, in turn, depends on (among other things) all past consumption of the unhealthy good. Although the current human capital stock does not affect agents' instantaneous utility directly, his/her current and past decisions regarding schooling affect human capital accumulation and, consequently, leisure-labor-school choices. In our model, the externality that the individual's current self imposes on his/her future selves is a two-dimension stock-externality, which is in line with standard models of human capital and in

Table V: Heterogeneous agents: optimal unconstrained second-best policies

		Economy				
		V	VI	VII	VIII	IX
Allocations	c^{ij*}					
	x^{ij*}					
	e^{ij*}					
	l^{ij*}					
	s^{ij*}					
	m^{ij*}					
	k^{ij*}					
	h^{ij*}					
Policies	S^*					
	H^*					
	M^*					
	T^*					
	O^*					
	T^*					
Welfare	U^{ij*}					
	U^{P*}					

health economics. In the Ramsey optimal taxation tradition, we show that the policy package that implements the social optimum contains subsidies directed to wealth and health and human, either separately or jointly through their effect on an agent's labor earnings. The optimal policy set does not contain a tax on the unhealthy consumption or a subsidy on the flow of resources spent to improve an individual's health. We further explore how a paternalistic optimal policy must not only take into account agents' self-control problems but also potential interactions of such (lack of) skills and cognitive skills.

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Appendix

TBA

Proof Proposition 1

Rewrite the type ij household budget constraint

$$(1+R_{t+1}-\delta_k)\left(1+S^{ij}_{t+1}\right)k^{ij}_{t+1}+W_{t+1}A^{ij}_{t+1}+H^{ij}_{t+1}h^{ij}_{t+1}+M^{ij}_{t+1}m^{ij}_{t+1}-T^{ij}_{t+1} \quad = \quad c^{ij}_{t+1}+x^{ij}_{t+1}+e^{i}{}_{t+1}+k^{ij}_{t+2}$$

and the government budget constraint

$$(1+R_{t+1}-\delta_k)\,S^{ij}_{t+1}k^{ij}_{t+1}+H^{ij}_{t+1}h^{ij}_{t+1}+M^{ij}_{t+1}m^{ij}_{t+1} \quad = \quad T^{ij}_{t+1}$$

for any agent type ij and period $t+1$.

Proof Proposition 2

It is straightforward and follows the same steps as the proof of Proposition 1.

Proof Proposition 3

TBA

Proof Proposition 4

It is straightforward and follows the same steps as the proof of Proposition 3.

Second-Best problem in Lagrangian form

$$
\begin{aligned}
\mathscr{L}_P^{2nd} &= \sum_{t=0}^{\infty} \Theta^t \left\{ \sum_{i,j} \gamma^{ij} \left[u\left(c_t^{ij}, x_t^{ij}, m_t^{ij}\right) + v\left(z_t^{ij}\right) \right] \right. \\
&+ \sum_{t=0}^{\infty} \sum_{i,j} \Theta^t \gamma^{ij} \psi_t^{ij} \left[u_x^{ij}(t) - u_c^{ij}(t) + u_c^{ij}(t) \frac{g_x^{ij}(t)}{g_e^{ij}(t)} \right] \\
&+ \sum_{i,j} \gamma^{ij} \widehat{\psi}_0^{ij} \left(u_c^{ij}(0) - \beta_j \Theta u_c^{ij}(1) \left(1 + R_1 - \delta_k\right) \left(1 + \widetilde{S}_1^*\right) \right) \\
&+ \sum_{s=t+1}^{\infty} \sum_{i,j} \Theta^{s-t} \gamma^{ij} \widehat{\psi}_s^{ij} \left(u_c^{ij}(s) - \beta_j \Theta u_c^{ij}(s+1) \left(1 + R_{s+1} - \delta_k\right) \left(1 + \widetilde{S}_{s+1}^*\right) \right) \\
&+ \sum_{t=0}^{\infty} \sum_{i,j} \Theta^t \gamma^{ij} \widetilde{\psi}_t^{ij} \left[v_z^{ij}(t) - u_c^{ij}(t) W_t m_t^{ij} h_t^{ij} \right] \\
&+ \sum_{i,j} \gamma^{ij} \omega_0^{ij} \left[\Theta \left[u_m^{ij}(1) + u_c^{ij}(1) \left(W_1 h_1^{ij} l_1^{ij} + \widetilde{M}_1^* \right) + \left(1 - \delta_m\right) \frac{u_c^{ij}(1)}{g_e^{ij}(1)} \right] - \frac{u_c^{ij}(0)}{g_e^{ij}(0)} \right] \\
&+ \sum_{s=t+1}^{\infty} \sum_{i,j} \Theta^{s-t} \gamma^{ij} \widehat{\omega}_s^{ij} \left(\beta^j \Theta \left[u_m^{ij}(s+1) + u_c^{ij}(s+1) \left(W_{s+1} h_{s+1}^{ij} l_{s+1}^{ij} + \widetilde{M}_{s+1}^* \right) \right. \right. \\
&\quad \left. \left. + \left(1 - \delta_m\right) \frac{u_c^{ij}(s+1)}{g_e^{ij}(s+1)} \right] - \frac{u_c^{ij}(s)}{g_e^{ij}(s)} \right) \\
&+ \sum_{i,j} \gamma^{ij} \sigma_0^{ij} \left[\Theta \left[u_c^{ij}(1) \left(W_1 m_1^{ij} l_1^{ij} + \widetilde{H}_1^* \right) + \left(1 - \delta_h\right) \frac{\zeta^i v_z^{ij}(1)}{B_s^{ij}(1)} \right] - \frac{\zeta^i v_z^{ij}(0)}{B_s^{ij}(0)} \right] \\
&+ \sum_{s=t+1}^{\infty} \sum_{i,j} \Theta^{s-t} \gamma^{ij} \widehat{\sigma}_s^{ij} \left(\beta^j \Theta \left[u_c^{ij}(s+1) \left(W_{s+1} m_{s+1}^{ij} l_{s+1}^{ij} + \widetilde{H}_{s+1}^* \right) \right. \right. \\
&\quad \left. \left. + \left(1 - \delta_h\right) \frac{\zeta^i v_z^{ij}(s+1)}{B_s^{ij}(s+1)} \right] - \frac{\zeta^i v_z^{ij}(s)}{B_s^{ij}(s)} \right) \\
&+ \sum_{i,j} \gamma^{ij} \lambda_0^{ij} \left[W_0 \left(m_0^{ij} h_0^{ij} \right) l_0^{ij} + \left(1 + R_0 - \delta_k\right) k_0^{ij} - c_0^{ij} - x_0^{ij} - e_0^{ij} - k_1^{ij} \right] \\
&+ \sum_{s=t+1}^{\infty} \sum_{i,j} \Theta^{s-t} \gamma^{ij} \lambda_t^{ij} \left(W_s \left(m_s^{ij} h_s^{ij} \right) l_s^{ij} + \left(1 + R_s - \delta_k\right) \left(1 + \widetilde{S}_s^*\right) k_s^{ij} \right. \\
&\quad \left. - c_s^{ij} - x_s^{ij} - e_s^{ij} - k_{s+1}^{ij} + \widetilde{H}_s^* h_s^{ij} + m_s^{ij} \widetilde{M}_s^* \right) \\
&+ \sum_{s=t+1}^{\infty} \sum_{i,j} \Theta^{s-t} \gamma^{ij} \eta_t \left[-\widetilde{S}_s^* \left(1 + R_s - \delta_k\right) k_s^{ij} - \widetilde{H}_s^* h_s^{ij} - m_s^{ij} \widetilde{M}_s^* + \widetilde{T}_s^* \right]
\end{aligned}
$$

$$
+ \sum_{t=0}^{\infty} \sum_{i,j} \Theta^t \gamma^{ij} \widehat{\eta}_t^{ij} \left[h_{t+1}^{ij} - \left(1 - \delta_h\right) h_t^{ij} - B\left(s_t^{ij}\right) \right]
$$