Fiscal Adjustments and the Shadow Economy in an Emerging Market*

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Abstract

We build an open-economy DSGE model that allows us to: (i) derive a time series for labor informality in Brazil spanning the period 2004-2018, whose evolution is consistent with the behavior of the main series provided by *Pesquisa Nacional por Amostra de Domicílios* (PNAD); (ii) run dynamic simulations showing that, in the presence of a large informal labor market (around 50% of the total labor force), expenditure-cutting measures lead, at worst, to mild short-run recessions in the formal sector and are likely to foster public debt sustainability. Likewise, adjustments through some kinds of distortionary taxation, mainly the corporate tax, and to a lesser extent, the consumption tax, also seem to improve both public debt dynamics and fiscal collection without a significant cost in terms of output. Thus, in countries with large informal economies such as Brazil, expenditure-based consolidations, as well as some sorts of tax-based adjustments, should be relied upon when trying to put the fiscal house in order.

Key-words: DSGE models, shadow economy, fiscal policy, New Keynesian model, emerging economies.

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1 Introduction

Irrespective of the level of development, if there is a feature all countries unambiguously share, that is the existence of the underground economy[1]. The ratio of the latter variable to GDP varies significantly across countries, being typically higher in developing and emerging economies than in the industrialized world[2]. For instance, Schneider *et al.* (2010) report the average size of the informal economy for 162 countries over 1999-2007. Expectedly, Latin America and the Caribbean top the list with 41.4% of GDP, closely followed by Sub-Saharan Africa, 40.2%, and then by Eastern Europe and Central Asia, with the same percentage, 38.9%. In OECD countries, informality accounts for a considerably lower share of GDP, 17.1%.

This phenomenon has clearly drawn a lot of attention of politicians and academics alike, and it has consequently sparked a great deal of research. Discussions on this matter intensify every time the economy finds itself plagued with high unemployment and significant fiscal imbalances. The negative economic consequences of having a sizable unofficial economy are several-fold: Fiscal deficits and debts are likely to be higher than they would otherwise be in the absence of large informal sectors, crowding-out private investment and leading to lower long-term growth through that channel (Elgin and Uras, 2013). In addition, a notable informal economy can lead to both greater financial instability and sovereign risk (Elgin and Uras, 2013). Furthermore, it may give rise to a lower quality and/or amount of public goods provided by the government (Loayza, 1996; Johnson *et al.*, 1997; Dessy and Pallage, 2003). Another possible shortcoming would be that it can bring about permanent adverse effects on total factor productivity, as informal firms and workers usually face greater barriers in the access to credit and training than their official counterparts (Loayza and Rigolini, 2011; Leal-Ordóñez, 2014). Against this background, squeezing the shadow economy seems like the right path to take.

On the other hand, the Great Recession might have somehow helped change the way economists and analysts think about the informal economy. Actually, the dire straits in which many economies have recently found themselves have made many reconsider to some extent the idea that the underground economy is a drag on economic prosperity and a signal of economic dysfunctionality. Absent these informal sectors, this global crisis would have likely had harsher implications for the standard of living and poverty worldwide. This is especially true for underdeveloped (developing and emerging) economies. As indicated above, many countries included in this category tend to have large informal sectors which have played a crucial buffer role in absorbing idle labor and capital in the aftermath of the financial crisis. In this respect, a recent article that studies this cushioning role of the underground sector in the wake of banking crises is Colombo *et al.* (2016).

In this work we aim to shed some additional light on this phenomenon by contributing to the existing literature in two different ways: first, we estimate the size of informality in Brazil's labor market using a two-sector –official versus unofficial– open-economy

[1]While they may not mean exactly the same, in this paper we follow a strand of the literature that uses different names interchangeably to refer to the same phenomenon: underground economy, hidden economy, shadow economy, unofficial economy, informal economy, black market economy, grey economy, unregistered economy, unobserved economy, etc. Our definition of this variable accords well with the one provided by the OECD (2002), for which the informal economy consists of legal activities that are considered to be productive in an economic sense, but that are concealed from the fiscal authorities so as to avoid paying taxes, being subject to labor legislation, etc. For a general overview on the shadow economy, see for example Schneider and Enste (2013).

[2]See for example Buehn and Schneider (2012).

DSGE model with price and wage frictions. The quarterly time series we derive is compatible with the information provided by PNAD's series[3]. Secondly, by conducting some simulation-based experiments through the same model, we are able to show that the level of the shadow economy (measured as the share of informal employment in total employment) makes a difference for the effect of fiscal adjustments on public debt dynamics and on the macroeconomy overall. Indeed, cutting any type of government expenditure in economies with high levels of informality –50% of unregistered employment over total, as typically found in low-income and emerging markets– unequivocally improves public debt sustainability and need not lead to an austerity-induced recession, at least not to a deep and protracted one, because of the role of the underground sector as a "shock absorber". By the same token, for the same kind of economies, adjusting through higher tax rates on consumption or on the firm's revenue leads to a reduction in the debt-GDP ratio and to a rise in tax collection without a big cost in terms of output. Barring lump-sum taxation, the remaining forms of revenue-based consolidations yield mixed results. Thus, in times of fiscal distress, countries whose ratio of informal to total employment hovers around 50% are well advised to rely on public spending cuts and on increases in some sort of taxation when trying to curtail fiscal deficits and bring the growth of public debt to GDP ratio to a halt.

It should be highlighted that these results differ from the ones we find when the same benchmark economy (Brazil) is calibrated to lower steady-state levels of informal employment –15%–, figures often seen in more industrialized countries. Regarding expenditure-based fiscal adjustments and some sort of tax-based adjustments, their positive effects on public debt sustainability are milder and more short-lived than for the economy with high informality referred to above. As for fiscal consolidation programs relying on the remainder of tax instruments, economies with fewer informal workers should expect their public debt to GDP ratios to deteriorate less over time than in the case of an economy with a large shadow economy (measured in terms of employment). In addition to this introduction, the paper is organized as follows: section 2 reviews the related literature, section 3 describes the model, section 4 presents and analyzes the results and section 5 concludes.

2 Literature review

The idea of employing dynamic general equilibrium models to analyze the informal economy has been explored before in the literature. More than a decade ago, Busato and Chiarini (2004) embedded an informal sector into an otherwise standard real business cycle (RBC) model to find that its performance improved along several dimensions. More recently, Restrepo-Echavarria (2014) made use of an open economy RBC model to show that the reason why consumption volatility amply exceeds output volatility is that economic agents are able to substitute out formal-good consumption for informal-good consumption in response to a productivity shock –a fact widely observed in developing countries and in some developed ones.

Two papers strike us as the most similar ones to ours regarding the objectives they pursue on the fiscal adjustment analysis: Pappa *et al.* (2015) and Annicchiarico and Cesaroni

[3]The National Household Survey (*Pesquisa Nacional por Amostra de Domicílios* –PNAD, in Portuguese–) is a survey conducted by the Brazilian Institute of Geography and Statistics (*Instituto Brasileiro de Geografia e Estatística* –IBGE, in Portuguese–) seeking to explore Brazil's socio-economic features.

(2018). The former authors use a New-Keynesian model featuring corruption and fiscal evasion to examine the effects of expenditure-based and revenue-based fiscal consolidations on economic activity in the European periphery. According to them, slashing government expenditures dampens the output losses and leads to a permanent reduction in the unemployment rate and significant welfare gains, whereas tax spikes exacerbate recessions, due to the fact that consumption, investment and labor participation drop in a meaningful way. The latter paper resorts to an open-economy New-Keynesian model calibrated to the Italian economy to study the relationship between growth-friendly deficit-neutral tax reforms and resource reallocations between the formal and the informal sectors. They find that these tax changes, besides having an overall positive effect on GDP and employment through the usual channels, trigger reallocation of labor and capital toward the relatively more productive sector, the formal one, which tends to have a positive impact on economic activity. Our article distinguishes itself from the above ones in that it attempts to account for the effects of fiscal adjustments in an emerging market with varying degrees of informality.

Along the same lines, concerning the estimated series of the informal economy, two articles resemble ours: Orsi *et al.* (2014) construct a DSGE model that generates an upward trend time series for the Italian shadow economy that lends support to the authors' thesis of an ever-growing tax burden as the ultimate cause of this phenomenon. In addition, they show that by lowering taxation and/or improving tax-enforcement, informality would drop, tax collection would rise, and overall economic activity would expand; in a similar vein, Argentiero and Bollino (2015) come up with the average steady-state size of the Italian unofficial economy over the period 1974:01-2011:02 (20% of GDP[4]) by means of a three-sector DSGE model containing regular, underground and criminal firm-related activities. As in Orsi *et al.* (2014), they find that high taxation and weak tax-enforcement can account for the relatively high importance of the informal sector in Italy. We reckon that our model, which features a small open economy with price, wage, and financial frictions, is likely to do a more efficient job of estimating the size of the underground economy.

Turnovsky and Basher (2009) address the so-called "recursive fiscal dilemma", by which low-income countries suffering from structural informality do not succeed in raising more revenue by taxing more the private sector because these increases in taxation encourage the reallocation of resources from the formal to the informal sectors, thereby offsetting in the end the efforts of the fiscal authorities to collect more revenues. Chatterjee and Turnovsky (2018) use a dynamic general equilibrium model to explore a channel of great importance for many developing countries: the link between remittance inflows and the size of the underground economy. They find that permanent inflows of such an external transfer bring about a short-run output expansion but contract economic activity (both economy-wide and in the formal sector) in the long run through a real exchange rate appreciation ("Dutch-Disease")[5], whereas temporary positive shocks to remittances only affect overall output and the informal sector negatively in the short-run. However, the very existence of strong collateral effects could reverse these negative effects, on the grounds that permanent shocks to remittances would avert the long-run Dutch-Disease phenomenon, thereby leading to a decline in informality and an increase in output, as well as temporary inflows would raise formal economic activity and total production in the short-run.

[4]This estimated number is 2 percentage points higher than the *Istituto Nazionale di Statistica* (ISTAT)'s estimation in 2010.

[5]For a thorough examination on the relationship between remittances and the Dutch Disease, see also Acosta *et al.* (2009).

By adopting a general equilibrium microfounded model, Prado (2011) analyzes the process whereby better-performing firms (those with higher productivity) find it advantageous to operate in the formal sector, whereas small firms prefer to go into informality. In addition, this author finds that gains in terms of welfare and higher output can be significant when reforms of the level of enforcement and the number and quality of regulations are undertaken. Leal-Ordóñez (2014) examines the distortions stemming from the combination of existing large informal sectors and incomplete tax enforcement and comes up with an inverted-U relationship between output and the underground sector. That is to say, in countries with small informal sectors, improving enforcement leads to higher output, while the opposite occurs in countries with large informal sectors. Using a two-sector growth model, Elgin and Solis-Garcia (2015) tackle the negative relationship between the tax burden and the underground economy to which some empirical studies point. They find that factors like the level of tax enforcement, productivity in the formal sector and physical capital depreciation are likely to play an important role in accounting for this negative relationship. Dell'Anno (2018) aims to investigate the link between income distribution and informality by way of an open-economy overlapping generation model that accounts for imperfect credit markets, indivisible entry costs to start a formal firm, and differing preferences of self-employed workers over going to the formal or informal sector. He finds a steady-state inverted-U relationship between inequality and informality: equal countries should be expected to have low levels of informality, whereas highly unequal countries would be more likely to have large underground sectors.

3 The model

This section develops a DSGE model featuring a two-sector open economy with price and wage stickiness[6], habit persistence and credit-constrained firms in the informal sector. Unlike what has so far been customary in DSGE models with informality, we discriminate between both types of labor markets as an attempt to capture the fact that some labor institutions (like labor courts, unions or minimum wages) may play an important role as "barriers to entry" in the official market but not in the unofficial one. When it comes to the source of the informal economy, however, we take the standard route largely followed in the literature: informal activities arise from the optimizing behavior of the representative agents. In this sense, households decide either to work in the shadow economy in order to circumvent the payment of labor-income taxes, but at the cost of giving up on most of the social benefits they would otherwise be entitled to, or to work in the official sector, which implies having to comply with their tax obligations but being able to enjoy the welfare state. On the labor-demand side, the representative firm faces the following decision: it has to choose whether it produces using inputs from the regular sector, in which case it is subject to a tax on its revenues and to a social security contribution or, conversely, it conceals its activity by producing in the underground economy. This latter alternative renders the firm's revenues untaxed but this agent will be monitored with some regularity and if it is caught incurring tax evasion, it will be compelled to pay the due tax plus a fine.

Empirical evidence supporting the fact that individuals freely choose the sector – official versus unofficial– in which they operate can be found in some articles (Carneiro and Henley, 2001; Menezes-Filho *et al.*, 2004). These conclude that wage disparities across

[6] Although it bears stressing that only formal firms are subject to wage rigidity. Wages adjust instantaneously in the informal labor market.

sectors are strongly accounted for by workers' non-observable characteristics. Such results reinforce the idea that working in the informal market could be a desirable choice on the ground of inefficient labor laws, low levels of human capital, or some non-pecuniary characteristics and benefits attached to the informal jobs (Funkhouser, 1996; Marcoullier *et al.*, 1997; Maloney, 1998, 1999, 2004). The preceding findings are at odds with the theory of labor market segmentation, which posits a wage gap among similar individuals arising from the sector in which they work (Pero, 1992; Cacciamali and Fernandes, 1993; and Fernandes, 1996). In effect, there exists abundant evidence showing that wages tend to be lower in the unofficial economy (Pradhan and Van Soest, 1995; Maloney, 1998, 1999, 2004; Lehmann and Pignatti, 2007; Botelho and Ponczek, 2011, *inter alia*). This would be consistent with the existence of a segmented labor market owing to causes such as lack of formal jobs, mobility costs, certain practices that labor unions carry out, racial segregation and gender discrimination (Dickens and Lang, 1985; Ulyssea, 2006; Barros, 2015).

The upshot is then that, according to evidence-based knowledge, informality can originate from either optimizing behavior as well as from labor segmentation. This is in line with our approach in this work since our model assumes no barrier to entry to the informal sector, implying that it absorbs all the working force that does not find formal jobs, whether as salaried workers or as self-employed ones. On the other hand, there are several labor market institutions (minimum wages, labor courts, unions, etc.) prevailing in the market sector that give rise to adjustment costs and wage rigidity. We deal with these imperfections by modeling Calvo-type wage stickiness for the formal sector.

3.1 Households

3.1.1 Definition of consumption, saving and informal work

In the model there is a continuum of infinitely-lived households indexed by $j \in [0,1]$. The stand-in household seeks to maximize its intertemporal welfare by choosing the consumption bundle, leisure and savings. Regarding this latter decision, the household is confronted with the choice over which instrument to use, physical capital versus financial assets (government, corporate and foreign bonds). As for the labor supply, this representative agent allocates the number of working hours between working in the market sector and working in the informal sector, contingent upon the existence of rigidities in the former sector which do not exist in the latter one. Hence, the stand-in household's maximization problem comes down to[7]:

$$\max_{C_{j,t},L^u_{j,t},I_{j,t},B_{j,t+1},B^F_{j,t+1},N_{j,t+1}} E_t \sum_{t=0}^{\infty} \beta^t S^P_t \left\{ \left[\frac{(C_{j,t} - \phi_c C_{j,t-1})^{1-\sigma}}{1-\sigma} \right] - S^L_t \left[\Omega_m \left(\frac{L^{m\,1+\psi_m}_{j,t}}{1+\psi_m} \right) + \left(\frac{L^{u\;1+\psi_u}_{j,t}}{1+\psi_u} \right) \right] \right\} \tag{1}$$

subject to the following budget constraint in each period

$$P_t(1+\tau^c_t)\left(C_{j,t} + I_{j,t}\right) + \frac{B_{j,t+1}}{R^B_t} + N_{j,t+1} + B^F_{j,t} R^F_{t-1} S_t$$

$$= (1-\tau^l_t) W^m_{j,t} L^m_{j,t} + W^u_{j,t} L^u_{j,t} + R_t K_{j,t} + B_{j,t} + R^N_{t-1} N_{j,t}$$

$$+ B^F_{j,t+1} S_t - \frac{\chi_{BF}}{2}\left(B^F_{j,t+1} - B^F_{j,ss}\right)^2 S_t - T_{j,t} \tag{2}$$

[7]The utility function used in this work follows Argentiero and Bollino (2015).

and to the standard law of motion for capital

$$K_{t+1} = (1-\delta)K_t + I_t \tag{3}$$

where E_t is the expectation operator; β is the intertemporal discount factor; ϕ_c is the habit formation parameter; ψ_m and ψ_u are the marginal disutility of labor in the market sector and the informal sector[8], respectively; $0 < \Omega_m < 1$ is a parameter capturing the better situation of formal workers relative to the informal ones as a result of the lack of access to social benefits in the underground economy; σ is the coefficient of relative risk aversion; C denotes consumption; L^m and L^u represent the amount of hours worked in the formal and informal markets, respectively; P is the price level; I refers to private investment; B is a one-year government bond whose rate of return is R^B; W^m and W^u are the levels of formal and informal wages, respectively; and K is the stock of capital with a rate of return R. The government's influence over the household decision-making process comes about through two distortionary taxes —a consumption tax, τ_t^c, and a labor-income tax, τ_t^l— and a non-distortionary one –a lump-sum tax, T; N is a corporate security issued by firms in the market sector, whose yield is R^N; B^F is a foreign bond, with a rate of return R^F and S is the nominal exchange rate. The term $\left[\frac{\chi_{BF}}{2}\left(B_{t+1}^F - B_{ss}^F\right)^2 S_t\right]$ is a stationarity-inducing technique (Schmitt-Grohé and Uribe, 2003).

We also consider two demand-side shocks in the model: S^P would be an intertemporal preference shock meant to capture short-run switches in consumer's tastes, whereas S^L would denote shocks to this agent's preferences toward leisure.

$$\log S_t^P = \rho_P \log S_{t-1}^P + \varepsilon_{P,t} \tag{4}$$

$$\log S_t^L = \rho_L \log S_{t-1}^L + \varepsilon_{L,t} \tag{5}$$

where ρ_P and ρ_L are the autoregressive components of these two shocks and where $\varepsilon_{P,t} \sim N(0, \sigma^P)$ and $\varepsilon_{L,t} \sim N(0, \sigma^L)$.

Solving the household's maximization problem yields the following first-order conditions:

$$\lambda_{j,t} P_t (1+\tau_t^c) = S_t^P \left(C_{j,t} - \phi_c C_{j,t-1}\right)^{-\sigma} - \phi_c \beta E_t \left[S_{t+1}^P \left(C_{j,t+1} - \phi_c C_{j,t}\right)^{-\sigma}\right] \tag{6}$$

$$\lambda_{j,t} W_{j,t}^u = S_t^P S_t^L {L_{j,t}^u}^{\psi_u} \tag{7}$$

$$\lambda_{j,t} P_t (1+\tau_t^c) = \beta E_t \left\{\lambda_{j,t+1}\left[(1-\delta)P_{t+1}(1+\tau_{t+1}^c) + R_{t+1}\right]\right\} \tag{8}$$

$$\frac{\lambda_{j,t}}{R_t^B} = \beta E_t \lambda_{j,t+1} \tag{9}$$

$$\frac{\lambda_{j,t}}{R_t^N} = \beta E_t \lambda_{j,t+1} \tag{10}$$

$$\lambda_{j,t} S_t \left[1 - \chi_{BF}\left(B_{j,t+1}^F - B_{j,ss}^F\right)\right] = R_t^F \beta E_t \lambda_{j,t+1} S_{t+1} \tag{11}$$

where λ is the Lagrange multiplier.

[8]It is reasonable to expect that $\psi_m < \psi_u$, for the marginal disutility of labor should be greater in the informal sector due to the lack of social protection.

3.1.2 Formal sector wage determination

Households' choice over the wage level involves the assumption that they supply differentiated labor under a monopolistically competitive framework. This service is sold to a representative labor-aggregating firm which combines all those different labor services into a single input by means of the following technology:

$$L_t^m = \left(\int_0^1 L_{j,t}^{m\ \frac{\varphi_{mt}-1}{\varphi_{mt}}} dj \right)^{\frac{\varphi_{mt}}{\varphi_{mt}-1}} \tag{12}$$

where $L_{j,t}^m$ is the amount of differentiated labor in the formal sector supplied by the household j, and φ_{mt} is the elasticity of substitution between differentiated labor inputs, subject to the following law of motion:

$$\log \varphi_{mt} = \rho_{\varphi_m} \log \varphi_{mt-1} + \varepsilon_{\varphi_m,t} \tag{13}$$

where ρ_{φ_m} is the autoregressive component of that shock, and $\varepsilon_{\varphi_m,t} \sim N(0, \sigma^{\varphi_m})$.

Provided the labor-aggregating firm operates under perfect competition, the aggregate wage level can be expressed as:

$$W_t^m = \left(\int_0^1 W_{j,t}^{m\ 1-\varphi_{mt}} dj \right)^{\frac{1}{1-\varphi_{mt}}} \tag{14}$$

with $W_{j,t}^m$ being the wage that each type of labor j receives.

Thus, this firm's demand for each differentiated labor j can be stated as:

$$L_{j,t}^m = L_t^m \left(\frac{W_t^m}{W_{j,t}^m} \right)^{\varphi_{mt}} \tag{15}$$

In setting the market sector wage level, the household seeks to solve the following problem:

$$\max_{W_{j,t}^m} E_t \sum_{i=0}^{\infty} (\beta\theta_W)^i \left\{ -S_{t+i}^P S_{t+i}^L \Omega_m \left(\frac{L_{j,t+i}^{m\ 1+\psi_m}}{1+\psi_m} \right) + \lambda_{j,t+i} \left[(1-\tau_{t+i}^l) W_{j,t}^m \ L_{j,t+i}^m \right] \right\} \tag{16}$$

subject to equation (15). Hence, the first-order condition for the previous problem is:

$$W_{j,t}^m \ = \left(\frac{\varphi_{mt}}{\varphi_{mt}-1} \right) E_t \sum_{i=0}^{\infty} (\beta\theta_W)^i \left[\Omega_m \frac{S_{t+i}^P S_{t+i}^L L_{j,t+i}^{m\ \psi_m}}{\lambda_{j,t+i}(1-\tau_{t+i}^l)} \right] \tag{17}$$

Notice that the labor market segmentation in the formal sector gives rise to a mark-up over the marginal rate of substitution between consumption and hours worked in that sector, something that is not seen in the informal sector.

Lastly, the aggregate wage level in the market sector is given by:

$$W_t^m = \left[\theta_W W_{t-1}^{m\ 1-\varphi_{mt}} + (1-\theta_W) W_t^{m\ \ 1-\varphi_{mt}} \right]^{\frac{1}{1-\varphi_{mt}}} \tag{18}$$

Proposition 3.1 (Informal labor supply decisions)**.** Let the combination of equations (7) and (17) constitute the criterion on which households base their labor supply decisions. The informal sector will be greater, the higher the relative wage in the aforementioned

sector $\left(\frac{W^u}{W^m}\right)$; the greater the markup of the formal wages $\left(\frac{\varphi_m}{\varphi_m - 1}\right)$; the greater the tax rate on labor income, τ^l; and the greater the number of households not allowed to change their wages, θ_W.

3.2 Firms

There are two types of firms in this economy: intermediate-good producers (wholesale), and final-good producers (retail). The former sector consists of a large number of firms, with each of them supplying differentiated goods. These wholesale firms face a two-fold decision: (i) the amount of inputs whether purchased in the market sector or in the shadow economy, to use in the production process, and (ii) the price of the good they will charge. By contrast, in the retail sector there is only a firm that, by employing a given technology, bundles the intermediate goods into a single good to be consumed by economic agents.

To fully achieve this task, this bundler buys a big amount of intermediate goods, which are used as inputs in the production process. It then follows that the retailer must maximize the following objective function:

$$\max_{Y_{j,t}} P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj \tag{19}$$

subject to the constraint:

$$Y_t = \left(\int_0^1 {Y_{j,t}}^{\frac{\varphi - 1}{\varphi}} dj \right)^{\frac{\varphi}{\varphi - 1}} \tag{20}$$

The first-order condition for the final-good producer's maximization problem can be written as:

$$Y_{j,t} = Y_t \left(\frac{P_t}{P_{j,t}} \right)^{\varphi} \tag{21}$$

and the general price level is:

$$P_t = \left(\int_0^1 {P_{j,t}}^{1-\varphi} dj \right)^{\frac{1}{1-\varphi}} \tag{22}$$

3.2.1 Intermediate goods-producing firm

This firm solves its problem in three steps. First, at the prevailing input prices and tax rates, it hires capital and labor in the official and unofficial sectors in order to minimize the total cost of producing domestic inputs. This step can in turn be split into two cost-minimization problems: one for the market sector and one for the informal sector[9].

[9]It should be noted that, as reported in Finkelstein Shapiro and Mandelman (2016), self-employment accounts for one-third to four-fifths of the labor force in developing and emerging markets, constituting one of the hallmarks of informal labor markets in these economies. For instance, in Central and Latin America, there is indeed a high correlation between self-employment and informal employment. For the sake of tractability, however, we refrain from including this feature into our model.

Market sector

Arguably, the government is able to track down all transactions made in the official sector. In this way, the wholesaler is faced with payment obligations such as payroll taxes, τ^s, and a corporate tax[10], τ^{corp}, but can also rely on credit (working capital) in order to pay wages[11], $(1+\tau_t^s)W_t^m R_t^f L_{j,t}^m$. Thus, the firm's minimization problem takes the following form[12]:

$$\min_{L_{j,t}^m, K_{j,t}} (1+\tau_t^s)W_t^m R_t^f L_{j,t}^m + R_t K_{j,t}^m + \tau_t^{corp} P_t^D INP_{j,t}^m \tag{23}$$

subject to the constraint:

$$INP_{j,t}^m = A_t^m {K_{j,t}^m}^{\alpha_1} {L_{j,t}^m}^{\alpha_2} {K_{j,G,t}^m}^{\alpha_3} \tag{24}$$

where $\left[R^f = \theta_f R^N + (1-\theta_f)\right]$ is the financial cost to the firm when relying on credit so as to pay wages, θ_f is the proportion by which the firm has recourse to this aforesaid source of funding to make those wage payments, INP^m is the input combination produced in the market sector, τ^s and τ^{corp} are, respectively, the payroll-tax rate and the corporate tax rate, K_G^m is the stock of public capital in the market sector, α_1, α_2 and α_3 are the shares of private capital, labor and public capital in the production of the domestic input, respectively[13], and A^m is the level of total factor productivity in the official sector[14], whose law of motion is:

$$\log A_t^m = \rho_{Am} \log A_{t-1}^m + \varepsilon_{Am,t} \tag{25}$$

where ρ_{Am} is the autoregressive component of this shock and $\varepsilon_{Am,t} \sim N(0, \sigma^{Am})$.

The first-order conditions for the previous problem are given by:

$$L_{j,t}^m = \alpha_2 \left(MC_t^m - \tau_t^{corp} P_t\right) \left[\frac{INP_{j,t}^m}{(1+\tau_t^s)W_t^m R_t^f}\right] \tag{26}$$

$$K_{j,t}^m = \alpha_1 \left(MC_t^m - \tau_t^{corp} P_t\right) \left(\frac{INP_{j,t}^m}{R_t}\right) \tag{27}$$

where MC_t^m is the firm's marginal cost in the market sector, and equations (26) and (27) represent the demands for labor and capital in the same sector.

Informal sector

While, as mentioned above, all domestic inputs produced in the formal sector are known by the fiscal authorities, this is not true for the case of the domestic inputs being produced in the unofficial sector. There, firms intentionally conceal their output for purposes of tax evasion. In each period, they face a probability (pr) of being inspected by the fiscal authority, so that if they are charged with a tax fraud crime, they will be required to pay the due tax plus a fine (s). Therefore, a firm operating in this sector seeks

[10] This tax enters the objective function as an additional cost.
[11] We follow Fuerst (1992), Carlstrom and Fuerst (1995), and Cooley and Quadrini (1999).
[12] The letter D stands for domestic.
[13] Without loss of generality, the value of these parameters will be identical in both sectors.
[14] It is taken to be different from the level of productivity in the informal sector.

to solve the following problem:

$$\min_{L^u_{j,t},K^u_{j,t}} W^u_t L^u_{j,t} + R_t K^u_{j,t} + pr_t s\tau^{corp}_t P_t INP^u_{j,t} \tag{28}$$

subject to the constraint:

$$INP^u_{j,t} = A^u_t {K^u_{j,t}}^{\alpha_1} {L^u_{j,t}}^{\alpha_2} {K^u_{j,G,t}}^{\alpha_3} \tag{29}$$

with the law of motion:

$$\log A^u_t = \rho_{Au} \log A^u_{t-1} + \varepsilon_{Au,t} \tag{30}$$

where ρ_{Au} is the autoregressive component of that shock and $\varepsilon_{Au,t} \sim N(0, \sigma^{Au})$.

As laid out before, we assume the existence of sector-specific technology shocks intended to capture potentially large intersectoral differences in labor productivity. This property is consistent with the empirical evidence, which points to a human capital gap in the informal labor market relative to the formal one (Marcelli *et al.*, 1999; Gallaway and Bernasek, 2002). More specifically, informal work tends to concentrate in low-productivity sectors, such as agriculture, construction and some service segments. On the contrary, the weight of informal labor is small in those high-productivity sectors, such as processing industries and, especially, extractive industries (mining) and financial intermediation. In this regard, Barbosa-Filho and Veloso (2016) provide figures for Brazil's agriculture sector and processing industry in 2013 that lend support to this evidence.

The probability of inspection by the fiscal authority follows an autoregressive process:

$$\log pr_t = \rho_{pr} \log pr_{t-1} + \varepsilon_{pr,t} \tag{31}$$

where ρ_{pr} is the autoregressive component of this inspection shock and $\varepsilon_{pr,t} \sim N(0, \sigma^{pr})$.

Solving the above minimization problem, we are left with the following first-order conditions:

$$L^u_{j,t} = \alpha_2 \left(MC^u_t - pr_t s\tau^{corp}_t P_t\right) \left(\frac{INP^u_{j,t}}{W^u_t}\right) \tag{32}$$

$$K^u_{j,t} = \alpha_1 \left(MC^u_t - pr_t s\tau^{corp}_t P_t\right) \left(\frac{INP^u_{j,t}}{R_t}\right) \tag{33}$$

where MC^u_t is the informal firm's marginal cost. Equations (32) and (33) denote, respectively, the demands for labor and capital in the unofficial sector.

The aggregation of the production of domestic inputs from both sectors gives:

$$INP^D_{j,t} = INP^m_{j,t} + INP^u_{j,t} \tag{34}$$

which can in turn be split into the domestic inputs used in the domestic production (INP^D_D) and those used in the rest of the world's production (INP^F_D):

$$INP^D_{j,t} = INP^D_{j,D,t} + INP^F_{j,D,t} \tag{35}$$

Proposition 3.2 (Informal labor demand decisions)**.** Let equations (26) and (32) determine the criterion upon which firms base their labor demand decisions in the underground economy. Informality will be greater, the higher the social security contribution, τ^s; the

lower the probability of being fined by the fiscal authority, pr; the higher the tax rate on the firm's revenues, τ^{corp}; and the higher the relative formal wage $\left(\frac{W^m}{W^u}\right)$.

Total cost and marginal cost

Thus far, the firm has been assumed to choose the amount of inputs that minimizes its cost (including both the formal and informal sectors). If we make the assumption that the output produced in both sectors is identical, $INP^m_{j,t}$ and $INP^u_{j,t}$ reduce to $INP^D_{j,t}$. Following the work of Busato and Chiarini (2004), the economy-wide total cost is just the sum of both sectors' total costs:

$$TC_t = \frac{INP^D_{j,t}}{A^m_t {K^m_{j,G,t}}^{\alpha_3}} \left[\frac{(1+\tau^s_t)W^m_t R^f_t}{(1-\alpha)}\right]^{1-\alpha} \left(\frac{R_t}{\alpha}\right)^{\alpha} + \frac{INP^D_{j,t}}{A^u_t {K^u_{j,G,t}}^{\alpha_3}} \left[\frac{W^u_t}{(1-\alpha)}\right]^{1-\alpha} \left(\frac{R_t}{\alpha}\right)^{\alpha} + (1+pr_t s)\tau^{corp}_t P_t INP^D_{j,t} \tag{36}$$

And the marginal cost ($MC = P^D$) follows this expression:

$$P^D_t = \left(\frac{1}{A^m_t {K^m_{j,G,t}}^{\alpha_3}}\right) \left[\frac{(1+\tau^s_t)W^m_t R^f_t}{(1-\alpha)}\right]^{1-\alpha} \left(\frac{R_t}{\alpha}\right)^{\alpha} + \left(\frac{1}{A^u_t {K^u_{j,G,t}}^{\alpha_3}}\right) \left[\frac{W^u_t}{(1-\alpha)}\right]^{1-\alpha} \left(\frac{R_t}{\alpha}\right)^{\alpha} + (1+pr_t s)\tau^{corp}_t P_t \tag{37}$$

where P^D is the price of the domestic input.

In a second stage, the firm chooses between inputs produced domestically (INP^D_D) and imported ones (INP^D_F) in order to produce the intermediate good Y_j, employing the following technology[15]:

$$Y_{j,t} = \left[(\omega_D)^{\frac{1}{\psi_D}} \left(INP^D_{j,D,t}\right)^{\frac{\psi_D - 1}{\psi_D}} + (1-\omega_D)^{\frac{1}{\psi_D}} \left(INP^D_{j,F,t}\right)^{\frac{\psi_D - 1}{\psi_D}}\right]^{\frac{\psi_D}{\psi_D - 1}} \tag{38}$$

where ω_D is the participation of the domestic input in the production of the intermediate good and ψ_D is the elasticity of substitution between domestic and imported inputs.

In this stage, the firm is known to solve the following problem:

$$\min_{INP^D_{j,D,t}, INP^D_{j,F,t}} INP^D_{j,D,t} P^D_t + INP^D_{j,F,t} S_t P^F_t \tag{39}$$

suject to the aforementioned technology (equation (38)), where P^F is the price of the imported input.

Solving this latter problem, one gets to the following first-order conditions:

$$INP^D_{j,D,t} = \omega_D \left(\frac{MC_{j,t}}{P^D_t}\right)^{\psi_D} Y_{j,t} \tag{40}$$

[15]This approach to modeling an open economy follows Castro *et al.* (2015)

and

$$INP^{D}_{j,F,t} = (1-\omega_D)\left(\frac{MC_{j,t}}{S_t P^F_t}\right)^{\psi_D} Y_{j,t} \tag{41}$$

with the marginal cost being equal to:

$$MC_{j,t} = \left[\omega_D {P^D_t}^{1-\psi_D} + (1-\omega_D)\left(S_t P^F_t\right)^{1-\psi_D}\right]^{\frac{1}{1-\psi_D}} \tag{42}$$

3.2.2 Calvo pricing

An intermediate goods-producing firm must set the price of its good according to the Calvo rule (Calvo, 1983). There is a probability θ that this wholesale firm keeps its price fixed in the next period and a probability $(1-\theta)$ of setting it optimally. Once the price has been set in period *t*, there is a probability θ that this price will remain fixed in period *t+1*, a probability θ^2 that this price will remain fixed in period *t+2*, and so on. Accordingly, this firm should take into account these probabilities when setting the price of its own good. The problem of the firm that adjusts the price of the good in period t is then:

$$\max_{P_{j,t}} E_t \sum_{i=0}^{\infty} (\beta\theta)^i Y_{j,t+i}\left(P_{j,t} - MC_{j,t+i}\right) \tag{43}$$

subject to equation (21).

After some rearrangement, the first-order condition of this maximization problem is given by:

$$P_{j,t} = \left(\frac{\varphi}{\varphi-1}\right) E_t \sum_{i=0}^{\infty} (\beta\theta)^i MC_{j,t+i} \tag{44}$$

Combining now the pricing rule (22) with the assumption that all price-changing firms set an equal price and that price-maintaining firms leave the price unaffected –since they share the same technology–, yields the overall final price:

$$P_t = \left[\theta {P_{t-1}}^{1-\varphi} + (1-\theta){P_t}^{\ 1-\varphi}\right]^{\frac{1}{1-\varphi}} \tag{45}$$

3.3 Government

In the model the government comes into the picture by splitting itself into two different entities: a fiscal authority and a monetary authority. The former is held responsible for conducting fiscal policy, while the latter pursues the price stability through a Taylor rule.

3.3.1 Fiscal authority

The government's budget constraint can be represented by:

$$\frac{B_{t+1}}{R^B_t} - B_t = P_t G_t + P_t I^m_{t,G} + P_t I^u_{t,G} - TAX_t - pr_t s\tau^{corp}_t INP^u_t P^D_t \tag{46}$$

where I^m_G and I^u_G are public investment in the market and the informal sectors, respectively.

And the tax collection would be:

$$TAX_t = \tau_t^c P_t(C_t + I_t) + \tau_t^{corp} IN P_t^m P_t^D + (\tau_t^l + \tau_t^s) W_t^m L_t^m + T_t \tag{47}$$

The law of motion for public capital in both sectors is:

$$K_{G,t+1}^m = (1-\delta) K_{G,t}^m + I_{t,G}^m \tag{48}$$

$$K_{G,t+1}^u = (1-\delta) K_{G,t}^u + I_{t,G}^u \tag{49}$$

The government avails itself of a number of fiscal instruments to achieve its goals. On the spending side, there would be current expenditure, G_t, public investment in the formal sector, $I_{t,G}^m$, and public investment in the underground sector, $I_{t,G}^u$. As for the revenue-generating tools, the government can resort to T_t, τ_t^c, τ_t^{corp}, τ_t^l and τ_t^s. All these instruments follow the same fiscal policy rule:

$$\frac{Z_t}{Z_{ss}} = \left(\frac{Z_{t-1}}{Z_{ss}}\right)^{\gamma_Z} \left(\frac{B_t}{Y_{t-1}P_{t-1}} \quad \frac{Y_{ss}P_{ss}}{B_{ss}}\right)^{(1-\gamma_Z)\phi_Z} S_t^Z \tag{50}$$

where $Z = \{G_t, I_{t,G}^m, I_{t,G}^u, T_t, \tau_t^c, \tau_t^{corp}, \tau_t^l, \tau_t^s\}$.

The fiscal shock can be given by:

$$\log S_t^Z = (1-\rho_Z) \log S_{ss}^Z + \rho_Z \log S_{t-1}^Z + \varepsilon_{Z,t} \tag{51}$$

where ρ_Z is the autoregressive component of this shock, and $\varepsilon_{Z,t} \sim N(0, \sigma^Z)$.

Finally, total tax evasion (TE) is given by:

$$TE_t = (\tau_t^l + \tau_t^s) W_t^u L_t^u + (1 - pr_t) \tau_t^{corp} IN P_t^u P_t^D \tag{52}$$

3.3.2 Monetary authority

The Central Bank's task is twofold: to foster output growth and to attain price stability. In order to accomplish this dual goal, it pursues a simple Taylor rule:

$$\frac{R_t^B}{R_{ss}^B} = \left(\frac{R_{t-1}^B}{R_{ss}^B}\right)^{\gamma_R} \left[\left(\frac{Y_t}{Y_{ss}}\right)^{\gamma_Y} \left(\frac{\pi_t}{\pi_{ss}}\right)^{\gamma_\pi}\right]^{(1-\gamma_R)} S_t^m \tag{53}$$

where γ_R is a parameter governing the stabilization of the movements in the interest rate and γ_Y and γ_π represent the sensibilities of the interest rate to output and to the inflation rate, respectively. S_t^m is the monetary shock, which abides by the following expression:

$$\log S_t^m = (1-\rho_m) \log S_{ss}^m + \rho_m \log S_{t-1}^m + \varepsilon_{m,t} \tag{54}$$

where ρ_m is the autoregressive component of that shock and $\varepsilon_{m,t} \sim N(0, \sigma^m)$.

Finally, the gross inflation rate can be defined as:

$$\pi_t = \frac{P_t}{P_{t-1}} \tag{55}$$

3.4 External sector (Rest of the world)

3.4.1 Foreign intermediate-good production

In the rest of the world's production process, a certain combination of inputs imported from the home country (INP_D^F) and of inputs produced internally (INP_F^F) is used:

$$\min_{INP_{D,j,t}^F, INP_{F,j,t}^F} INP_{D,j,t}^F P_t^D + INP_{F,j,t}^F S_t P_t^F \tag{56}$$

subject to

$$Y_{j,t}^F = \left[(1-\omega_F)^{\frac{1}{\psi_F}} INP_{F,j,t}^F{}^{\frac{\psi_F - 1}{\psi_F}} + \omega_F{}^{\frac{1}{\psi_F}} INP_{D,j,t}^F{}^{\frac{\psi_F - 1}{\psi_F}} \right]^{\frac{\psi_F}{\psi_F - 1}} \tag{57}$$

where Y^F is the foreign output and ω_F is the share of Brazilian imports in the rest of the world's output bundle.

The first-order condition for the above problem is:

$$INP_{D,j,t}^F = \omega_F \left(\frac{S_t P_t^F}{P_t^D} \right)^{\psi_F} Y_{j,t}^F \tag{58}$$

And the balance of payments constraint is given by:

$$S_t \left(B_{t+1}^F - R_{t-1}^F B_t^F \right) = S_t INP_{F,j,t}^D P_t^F - INP_{D,j,t}^F P_t^D \tag{59}$$

3.4.2 Shocks to income, interest rates and the input prices abroad

The law of motion of world income, the international interest rate and the price of imported inputs, respectively, are:

$$\log Y_t^F = \rho_{Y^F} \log Y_{t-1}^F + \varepsilon_{Y^F,t} \tag{60}$$

where $\varepsilon_{Y^F,t} \sim N(0, \sigma^{Y^F})$.

$$\log R_t^F = \rho_{R^F} \log R_{t-1}^F + \varepsilon_{R^F,t} \tag{61}$$

where $\varepsilon_{R^F,t} \sim N(0, \sigma^{R^F})$.

$$\log P_t^F = \rho_{P^F} \log P_{t-1}^F + \varepsilon_{P^F,t} \tag{62}$$

where $\varepsilon_{P^F,t} \sim N(0, \sigma^{P^F})$.

3.5 Equilibrium conditions

Lastly, in order to close the model, the following equilibrium conditions are needed:

Equilibrium condition in the goods market:

$$Y_t = C_t + I_t + G_t + I_{t,G}^m + I_{t,G}^u \tag{63}$$

Aggregate capital stock:

$$K_t = K_t^m + K_t^u \tag{64}$$

Aggregate labor:

$$L_t = L_t^m + L_t^u \tag{65}$$

From the above equations, it follows that a firm can produce its output using only inputs from the market sector. Consequently, inputs from the underground economy would not be strictly necessary in the production of final output. This means that in this model production in the informal sector occurs because firms are given the possibility of evading taxes by reallocating their total –or partial– output to the shadow economy.

4 Analysis of the results

4.1 Comparison of the estimated series of informality with those of PNAD

In this section we compare the estimated informality series with those provided by PNAD[16]. The results are displayed in Figure 1 and in Table 1. Figure 1 showcases a comparison between the series of informality estimated by the model (purple) and those obtained by PNAD through the direct method (definition I: red, definition II: blue, and definition III: green)[17]. From a simple visual inspection of that graphical comparison, it becomes apparent that the model-generated series shows a larger shadow economy at the beginning of the period which tends to fall faster and more abruptly than the PNAD series until 2012, when it hits a trough and bounces back strongly one year before the latter follow suit. The anticipation in the inflection point of the series detected by the model relative to the alternative statistical methods could be explained on the grounds that a radical change in the economic policy regime came about in 2011, when the President Rouseff was elected[18]. The Brazilian economy was hard hit in the aftermath of the Great Recession of 2008, but at that time, the government rightly reacted by stimulating the economy through expansionary fiscal and monetary measures. This stimulus package enabled the government to attain its goal of overcoming the downturn in just a few months. But even if this exceptional crisis had called for exceptional policies, abandoning them as

[16]PNAD employs the direct method so as to calculate three different measures of informality:

$$\text{Level of informality-definition I - (\%)} = \frac{A+B}{A+B+C}$$

$$\text{Level of informality-definition II - (\%)} = \frac{A+B+D}{A+B+C+D+E}$$

$$\text{Level of informality-definition III - (\%)} = \frac{A+B}{A+B+C+E}$$

where A are workers without labor contracts, B are self-employed workers, C are workers under a labor contract, D are unpaid workers and E are employers.

[17]The estimated series has been annualized using the mean of the quarterly values.

[18]See, for instance, Pastore (2015).

the economy resumed growing fast seemed to be sensible. This was not what happened. The government chose the option "prime the pump" despite the clear signs of increasing economic activity already present over the second semester of 2009. As mentioned above, the true regime switch occurred in 2011, when the government officially adopted a more interventionist economic model referred to as the "New Economic Matrix", targeted at promoting investment, for which two main instruments were used: selective subsidy schemes, which contributed to the swift and deep deterioration of the primary fiscal position; and an ultra expansionary monetary policy designed to lower the real interest rate so as to boost investment. Naturally, these two policy levers led to rising inflation, which prompted the government to implement a battery of price controls to fight it. After this brief narrative summary of the events, it does not seem surprising that forward-looking individuals responded to a higher expected tax burden, in addition to larger current inefficiencies generated by price controls, public subsidies and more cumbersome regulation, by shifting their activities toward the informal sector well before the PNAD series identified this turnaround.

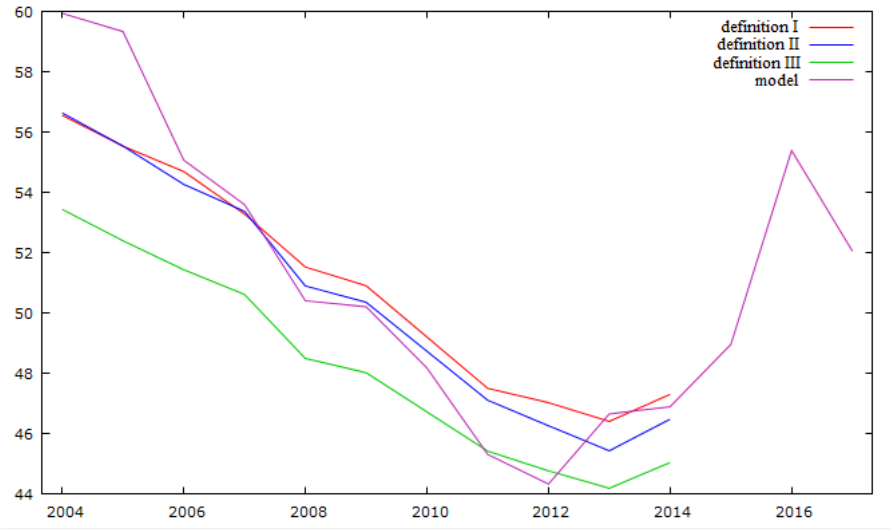

Figure 1: Comparison of the estimated series of informality with definitions I, II and III of informality provided by PNAD. Source: authors' calculations.

Table 1 displays the correlation coefficients of those series. Relative to the value associated with the estimated series, the results for the definitions I, II and III were, respectively, 0.9718, 0.9704 and 0.9738. Remarkably, all of them are well above the critical value for statistical significance at 95% level, 0.576.

Briefly, from the joint information supplied by Figure 1 and Table 1, it is safe to say that the model does a good job of estimating an informality series. This might constitute important evidence that microfoundations present in this class of models would play a relevant role in accounting for the behavior of the Brazilian shadow economy.

4.2 Fiscal adjustment in an informal economy

We next set out to analyze the macroeconomic effects of fiscal adjustments for different levels of informality. Toward this end, we run a counterfactual exercise whereby the

Table 1: Correlation coefficients over the time span considered. Source: authors' calculations.

DefinitionI	DefinitionII	DefinitionIII	Model	
1	0.9987	0.9986	0.9718	Definition I
	1	0, 9997	0.9704	Definition II
		1	0.9738	Definition III
			1	Model

Table 2: Descriptive statistics. Source: authors' calculations.

Variable	Mean	Median	Minimum	Maximum
Definition I	50.9003	50.9022	46.4013	56.5539
Definition II	50.4582	50.3521	45.4271	56.6316
Definition III	48.2272	48.0180	44.1840	53.4369
Model	51.1612	50.3038	44.3238	59.9332
Variable	Standard deviation	Coefficient of variation	Skewness	Kurtosis
Definition I	3.69449	0.0725830	0.224410	-1.42613
Definition II	4.00094	0.0792920	0.226795	-1.39603
Definition III	3.30561	0.0685424	0.273949	-1.38032
Model	4.94928	0.0967388	0.422276	-0.904546

emerging economy for which we calibrate and estimate our model, Brazil, can enjoy lower levels of unofficial activity in the labor market than it does in the real world. Clearly, to fully understand the interplay between fiscal austerity and the shadow economy, a benchmark economy with a smaller informal sector is needed. Hence, we study how fiscal consolidations play out under two different sizes of this variable: (i) developed economy (low level of labor informality, 15%), and (ii) developing/emerging economy (high level of informality in its labor market, 50%)[19]. The values of each shock used in this experiment are[20]: $\varepsilon_G = 0.1262$; $\varepsilon_T = 1.1972$; $\varepsilon_{\tau^c} = 0.2778$; $\varepsilon_{\tau^{corp}} = 0.6758$; $\varepsilon_{\tau^s} = 1.0026$; and $\varepsilon_{\tau^l} = 0.1682$. The main reason why running a quantitative test like this may be relevant is that currently a non-negligible proportion of the world's economies find themselves in situations of unprecedented fiscal distress, with high and rising debt to GDP ratios at a time when demographic factors are placing strong spending pressures on the public pension systems[21]. Brazil is no exception in this regard, whose public debt to GDP ratio is on an explosive trajectory, which calls for urgent fiscal consolidation measures. If governments are to accelerate, or even initiate, austerity programs, being fully aware of the effects of these fiscal adjustments on an economy with varying degrees of informality appears imperative.

The consensus in the literature underscores that expenditure-based plans, if perceived sustainable, can prompt transfers of inputs toward the formal sector, whereas the opposite occurs when tax hikes are resorted to –see, among others, Pappa *et al.* (2015). The underlying mechanism is straightforward: intuitively, public expenditure cuts crowd in private investment, which leads to an increase in the capital stock mainly in the official market (where investment is more profitable). This raises formal labor productivity, thereby further widening the productivity gap that already exists between these sectors. As far as tax increases are concerned, lower investment engineered by the greater distortions helps close the existing productivity divergence, which renders the underground economy relatively more attractive. Accordingly, capital and labor will flow out of the official market

[19]From here onwards, throughout this section we use the term developed economy to refer to an emerging economy, like Brazil, with a lower steady-state informal sector, as those seen in the industrialized world.

[20]The estimates are presented in the Appendix A.

[21]For a thorough analysis on the output and distributional effects of fiscal austerity in a context of high public debt and fiscal stress, see Glomm *et al.* (2018).

and into the informal sector. It is important to emphasize that by and large, our findings for an underdeveloped economy align themselves with this view, mainly with regard to expenditure-based consolidations, although we find that some tax-based adjustments also succeed in reining in, or even reducing, the public debt-to-GDP ratio.

Figure 2 plots the results of cutting government current expenditure. On impact, slashing current spending is found to cause a mild recession for the two levels of informality, albeit it is worth pointing out that the larger the shadow economy is, the smaller is the recessionary effect of the austerity policy. Output picks up quickly thereafter, entering positive territory as of the fifth quarter. A similar outcome is seen in the labor market. Remarkably, this policy significantly squeezes the size of the informal sector in the developing economy, something that does not happen in the benchmark economy. The ensuing reallocation of resources toward the formal sector referred to above ameliorates the direct negative impact of cutting spending. As far as the fiscal variables are concerned, the spending retrenchment is more beneficial in the presence of large underground economies, since tax revenues decline less and debt dynamics prominently improves.

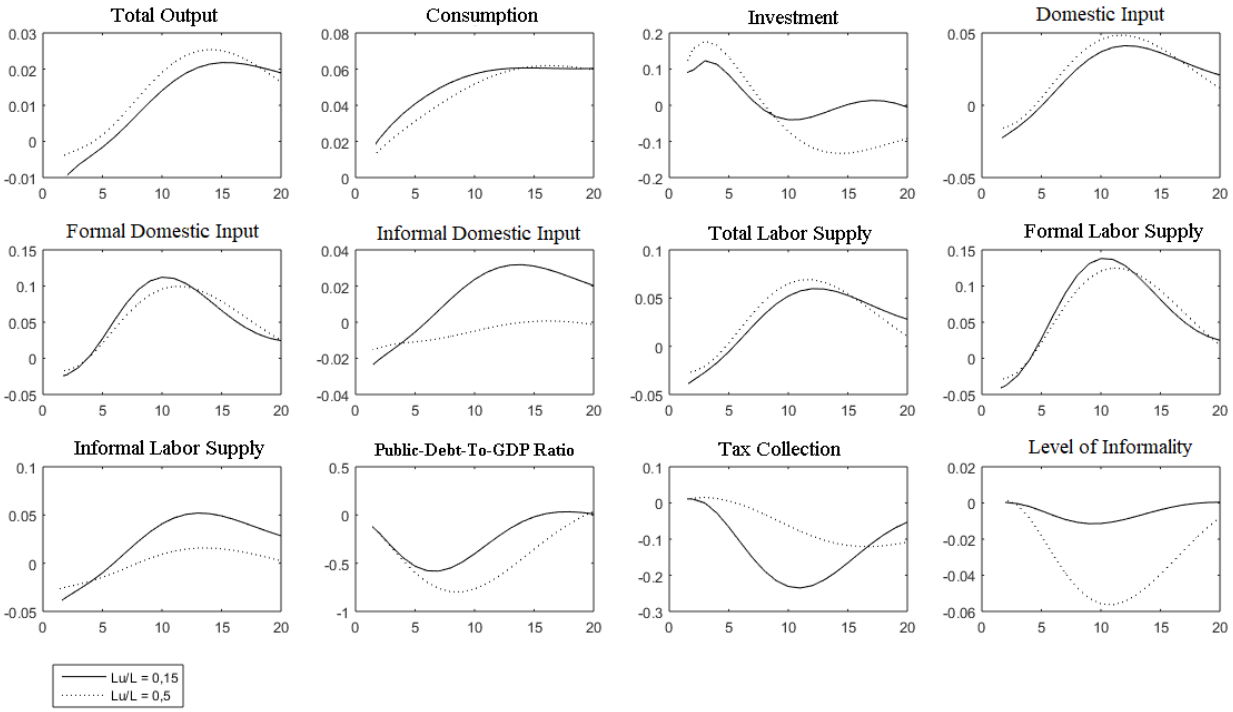

Figure 2: Government expenditure cut. Source: authors' calculations.

Figures 3 and 4 depict a reduction of the item public investment in the formal and informal sectors, respectively. A negative shock to public investment leads to qualitatively similar outcomes as those arising from cutting current spending, although the effects are more pronounced. Specifically, among the expenditure-based fiscal measures considered in our exercise, reducing government investment in the informal sector yields the largest decrease in the level of informality in the labor market and in the public debt-to-GDP ratio. Output– and consumption–wise, the economy with the sizable underground sector benefits the most, with private investment faring worse than in the benchmark economy.

A fiscal contraction based on higher lump-sum taxes is shown in Figure 5. With re-

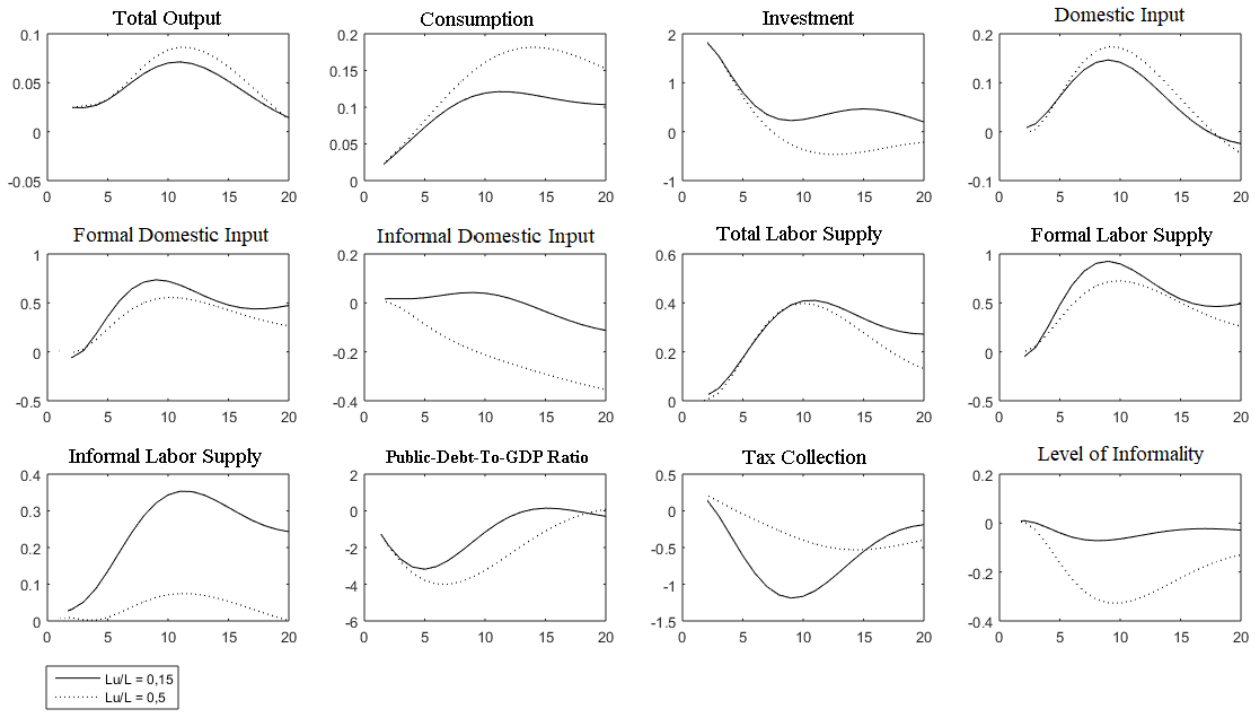

Figure 3: Decrease in public investment – formal sector. Source: authors' calculations.

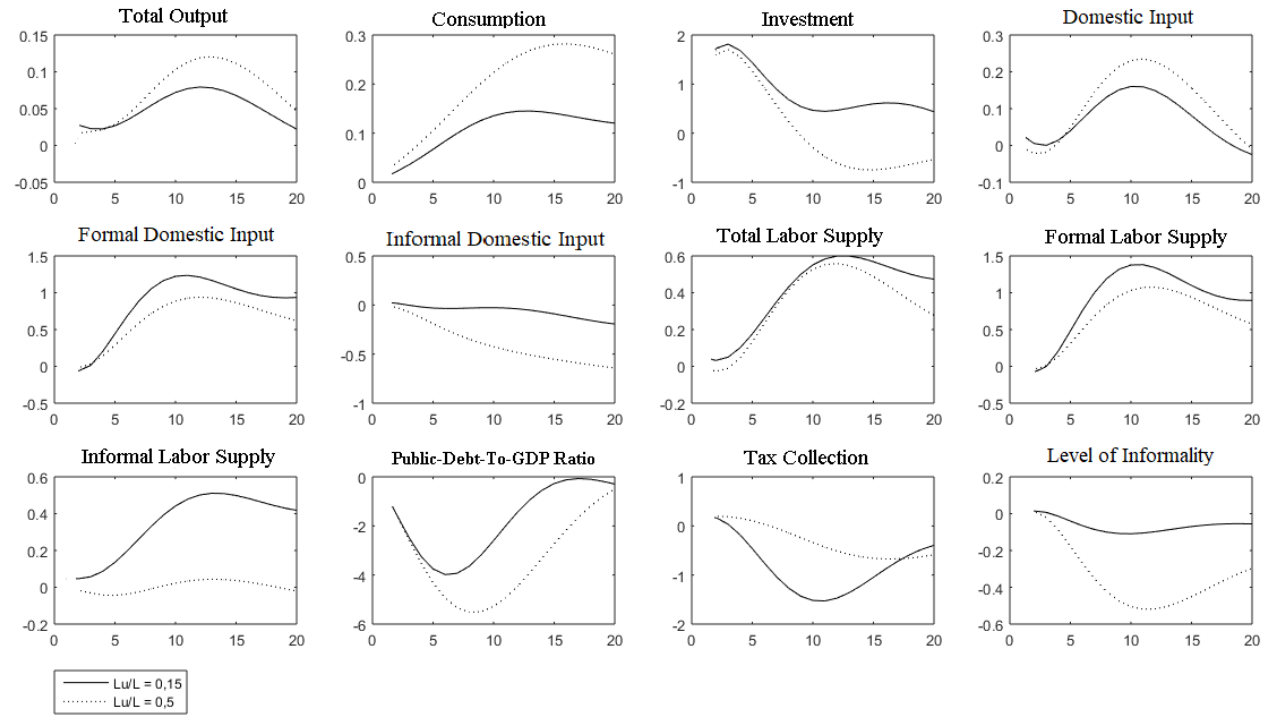

Figure 4: Decrease in public investment – informal sector. Source: authors' calculations.

gard to the developing economy, this restrictive fiscal policy immediately impairs total economic activity for almost 20 quarters and raises the level of informality for more than 15 quarters. Interestingly, all variables in the model evolve in the opposite way when we turn to the economy with low levels of informal activity. In addition, the fiscal position of this benchmark economy temporarily benefits more in relative terms from this measure, as both tax collection and public debt to GDP behave better for almost 10 quarters.

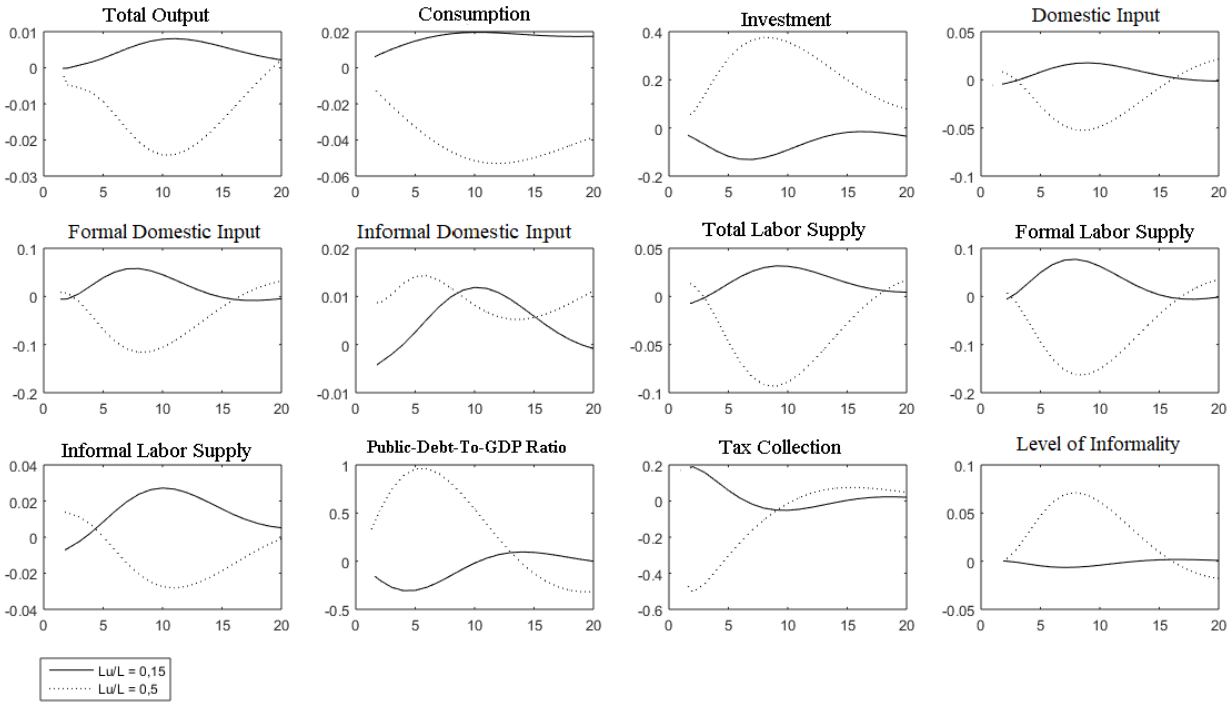

Figure 5: Increase in a lump-sum tax. Source: authors' calculations.

The results of a hike in the consumption-tax rate are given in Figure 6. The main divergence between both types of economies lies in the different way the level of informality reacts when the tax rate is raised. While there is a slight increase on impact, a higher indirect-tax burden thereafter causes informality in the developing economy to drop for several quarters until it changes course and ends up outpacing the level of this variable in the benchmark economy, which remains mostly unaltered over time. Besides, some of the short-run effects of this tax policy would be favorable in that total output would not get affected much and consumption would grow after some mild negative impact. However, investment would be lower for some quarters. On the fiscal front, the less developed economy performs better in relative terms, at least temporarily, for the evolution of the debt-to-GDP ratio and the tax collection compares favorably to the developed economy's. Therefore, in times of fiscal crisis, it would pay off for countries with large shadow economies to rely on consumption-tax increases on a temporary basis to combat budget deficits[22].

[22]A word of caution is in order here: our approach abstracts from income distribution considerations. An analysis covering the distributional effects of these fiscal measures should not ignore the fact that the burden of consumption taxes as a proportion of income is higher for the most vulnerable population groups. While this policy change would increase revenue and reduce the debt to GDP ratio without negative consequences for overall output, it could hurt the poor.

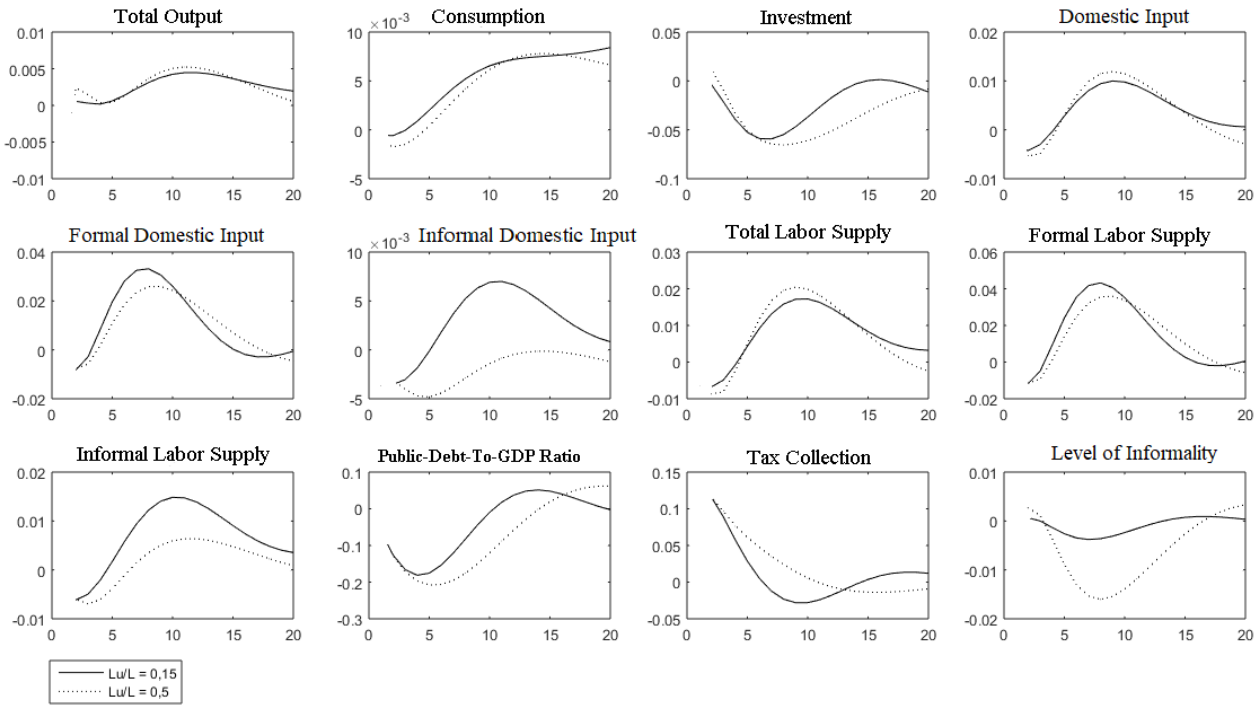

Figure 6: Increase in the consumption tax rate. Source: authors' calculations.

Figures 7 plots a fiscal adjustment implemented by means of a hike in the tax rate on the formal firm's revenue. This measure affects the firm's decision as to where to produce, in the formal versus the informal sector. Output is more affected in the benchmark economy, although this moderate negative effect ensues after some quarters. Likewise, in the presence of a small informal economy, consumption is severely hit by a higher corporate tax rate, which makes room for private investment to grow faster over time. The level of informality fluctuates more for an economy with a large underground sector, but the net effect of this tax policy becomes marginally unimportant approaching the tenth quarter. For this type of economies, raising the corporate tax enhances fiscal solvency significantly, as tax collection rises over the whole horizon. Given these results, a fiscal austerity program based on an increase in this tax is likely to be more successful in economies with high degrees of informality, since the fiscal side (tax collection and debt dynamics) would improve substantially without steering the economy into a recession.

Figures 8 and 9 exhibit a higher social security contribution borne by the firm and a higher labor-income tax, respectively. As these two measures deliver similar results, they will be analyzed jointly. The main difference between the two tax policies is the effect on consumption. The former acts to raise consumption for both sorts of economies, with this variable increasing noticeably more in the developed country. By contrast, the labor-income tax drives down consumption significantly in the economy with a large informal sector, but barely has an effect on it when we turn to the other economy. The level of informality experiences an ephemeral increase when both types of taxes are raised for the developing country, although it is worth stressing that the effect is more pronounced when the payroll taxes are used. When it comes to the fiscal side, the differences are small and short-lived across both types of economies. None of these tax policies improves debt sustainability and tax collection in a significant way.

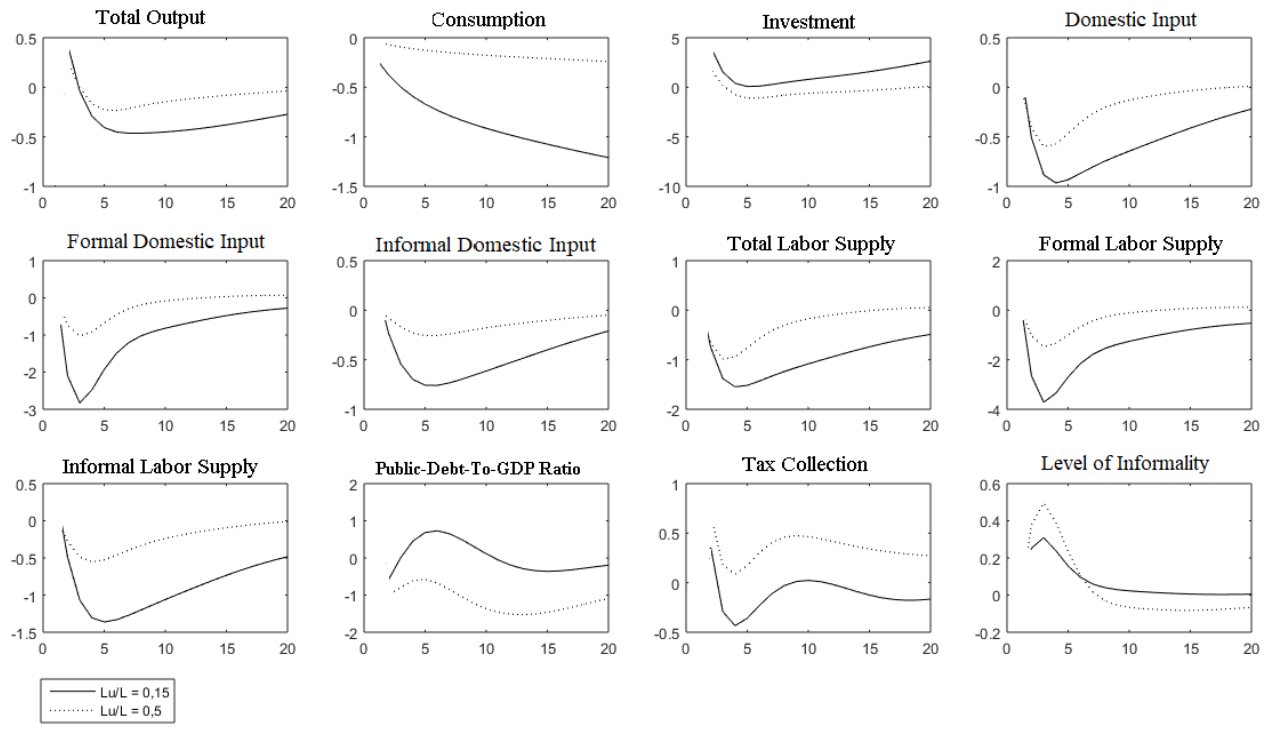

Figure 7: Increase in the tax rate on the formal firm's revenue. Source: authors' calculations.

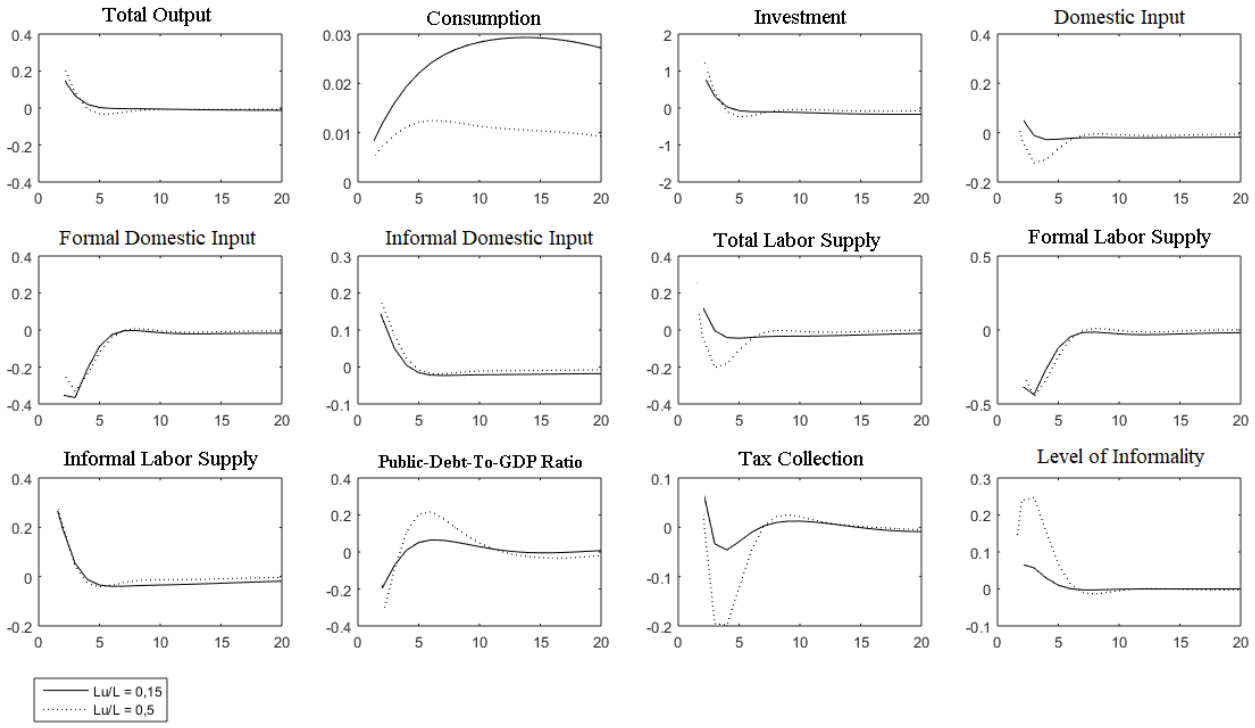

Figure 8: Increase in the social security contribution borne by the firm. Source: authors' calculations.

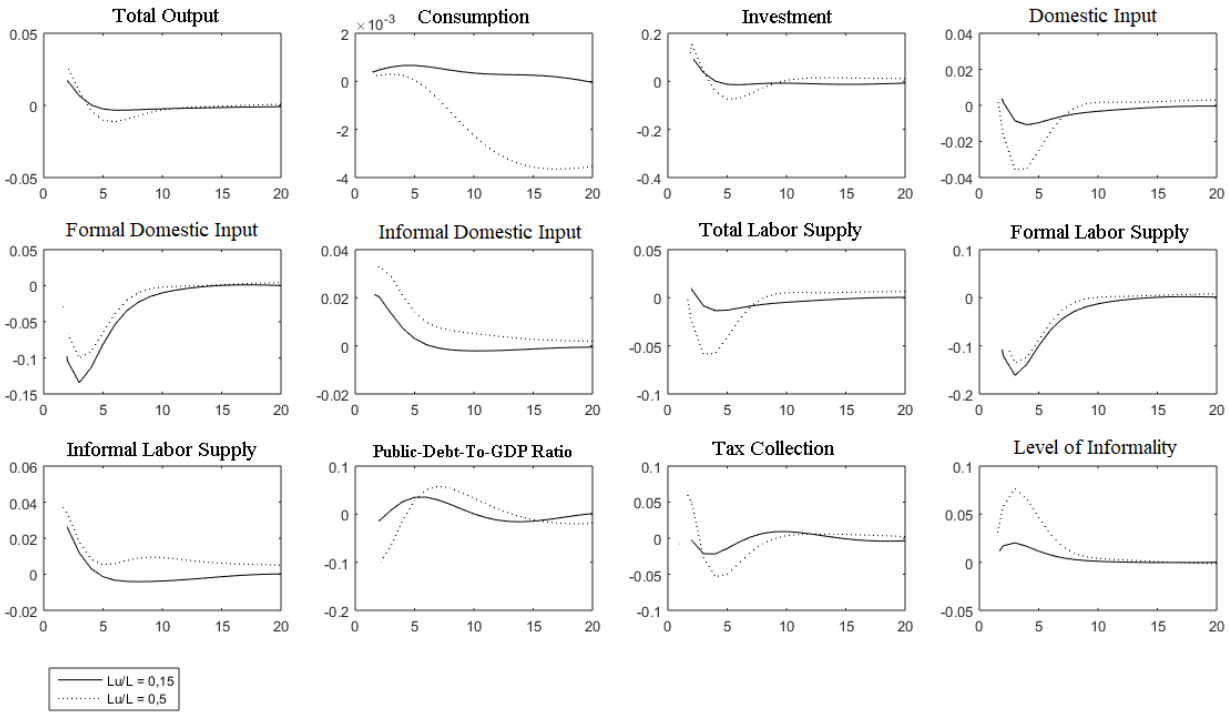

Figure 9: Increase in the tax rate on labor income. Source: authors' calculations.

5 Conclusions

In this work we intend to contribute to a better understanding of the interplay between fiscal shocks and informality and its ensuing effects on the macroeconomic variables in developing and emerging economies. With this purpose in mind, we first estimate a quarterly series of labor informality for the Brazilian economy through an open-economy DSGE model. The estimated series is consistent with the behavior of the main series provided by PNAD, although it should be noted that the model series exceeds the levels indicated by the direct methods initially, declines faster than them over the economic boom years and bounces back one year before the PNAD series do. We believe that DSGE models can be powerful tools for supplying accurate information of the evolution of this unobserved variable. Due to the dynamic microfounded structure, they have the potential to help understand the channels through which the underground economy is affected and they also allow making future predictions of this variable given the set of policies and shocks hitting the economy. Besides, this technique, when confronted with the direct methods, offers the added benefits of higher speed of implementation, lower cost and the very ability to provide data at a quarterly frequency.

Secondly, using the same model, we investigate which fiscal tools are both the less damaging in terms of economic performance and more effective at promoting public debt sustainability in these less developed economies facing fiscal crises. To provide a useful benchmark for our experiment, against the average size of the underground economy (in the labor market) usually seen in underdeveloped countries (around 50% of total labor force), we also consider a lower level of labor informality such as those corresponding to more developed economies (around 15%). The quantitative estimations that we conduct underline that cutting any type of government spending (current expenditure or public investment) when the informal labor market is very large (50% or above) need not be

that costly in terms of output losses on impact and may even spur formal economic activity over time. In addition, it leads to a drastic reduction in the level of informality, does not undermine tax collection and can improve debt dynamics. The findings differ when we calibrate the economy to a lower level of labor informality. On the contrary, barring the corporate tax and, to a lesser extent, the consumption tax, in these economies with high levels of informality tax-based fiscal consolidations would aggravate public debt sustainability and lower tax revenues, although this restrictive fiscal policy need not trigger an economic downturn due to the offsetting emergence of the underground economy. Therefore, less developed countries such as Brazil suffering fiscal woes should prioritize spending-cutting measures in their designed fiscal adjustment programs, notwithstanding the possibility that their governments can also rely on some tax instruments –the aforementioned corporate tax and the consumption tax– to help achieve their fiscal targets.

As a caveat, we should point out that policies seeking to formalize underground activities might prove ineffective if policy-makers do not tackle the deep-rooted causes whereby the elasticity of substitution of labor could be low. As laid out before, a low value for this variable is tied to a high degree of rigidity in the formal labor market. This is why policy-makers are suggested to take into consideration workers' and wages' heterogeneity when designing their fiscal policy packages. In the presence of a highly rigid formal labor market, social protection and inclusion policies should be targeted at low-wage workers.

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Appendix A: Structural estimation

This appendix contains the data processing procedure and the methodology for structural estimation.

Data processing

The model was estimated using quarterly date spanning 2003:Q1-2018:Q1 (sixty quarters). The observable variables were chosen on the ground that they could provide useful information in the estimation of the underground economy. Table 3 gives the sixteen observable variables used in this article. To process data, we employed the software X12-ARIMA. In addition, we included two measurement errors into the endogenized equations of the inflation rate –to compensate for the changes caused by administered prices–, and of the government tax collection –to compensate for the changes due to the simplification of the model on the fiscal side[23].

Table 3: Observable variables. Source: own elaboration.

Variable	Series	Source
C	Final consumption – Households – R$ (millions)	IBGE/SCN 2000 Trim.
G	Final consumption – Government – R$ (millions)	IBGE/SCN 2000 Trim.
I	Gross fixed capital formation – R$ (millions)	IBGE/SCN 2000 Trim.
L	Number of hours paid – Industry – Index (Jan. 2001 = 100)	PIMES/IBGE
π	IPCA (%)	IBGE/SNIPC
R^B	Selic Over – Key policy interest rate (%)	BCB Boletim/M. Finan.
TAX	Federal gross tax collection – R$ (millions)	Min. Fazenda/SRF
$\tau^l W^m L^m$	Income tax – R$ (millions)	Min. Fazenda/SRF
$\tau^{corp} Y^m P$	Corporate tax – R$ (millions)	Min. Fazenda/SRF
$\tau^c(C+I)P$	Consumption taxes – ICMS and IPI – R$ (millions))	Min. Fazenda/SRF
B	Domestic debt - public sector - net - (% GDP)	Bacen/Not. Imp./F. Pub
Y^F	Real Gross Domestic Product, Billions of Chained 2012 Dollars	Federal Reserve Bank of St. Louis
R^F	10-Year Treasury Constant Maturity Rate, Percent	Federal Reserve Bank of St. Louis
P^F	Consumer Price Index for All Urban Consumers: All Items, Index 1982-1984=100	Federal Reserve Bank of St. Louis
S	Exchange Rate - R$ / US$	Bacen / Boletim / BP
B^F	Foreign debt - public sector - net - (% GDP)	Bacen/Not. Imp./F. Pub

Calibrated parameters, prior and posterior

In this section of the Appendix A we pursue a two-tier approach in that some of the parameters not directly related to the main goal of this article are calibrated, while those relevant parameters for the analysis of the shock propagation are estimated using the Bayesian methodology. The main calibration procedure employed here is to pick the parameter values from other relevant articles in the DSGE literature. Table 4 summarizes the calibrated values for those parameters.

Given the prior distributions of the parameters, the model was estimated using a Markov chain process via the Metropolis-Hastings algorithm with 100,000 iterations and 10 parallel chains. The results of the Bayesian estimation are shown in Table 5 and in

[23]We adopt the similar procedure as in Castro *et al.* (2015).

Table 4: Calibrated parameters. Source: own elaboration.

Parameter	Value	Source
σ	2	Cavalcanti and Vereda (2011)
β	0.985	Cavalcanti and Vereda (2011)
δ	0.025	Cavalcanti and Vereda (2011)
ω_D	0.15	IBGE/SCN 2000
ω_F	0.02	IBGE/SCN 2000
ϕ_c	0.74	Castro *et al.* (2015)
χ_{BF}	-0.003	sensitivity analysis
P_{ss}	4	sensitivity analysis
P_{ss}^F	6	sensitivity analysis
α_1	0.6	Mussolini (2011)
α_2	0.3	Mussolini (2011)
α_3	$0.4 - \alpha_1$	-
pr_{ss}	0.03	Orsi *et al.* (2014)
s	1.3	Orsi *et al.* (2014)
φ	8.8	Castro *et al.* (2015)
θ_W	0.75	Castro *et al.* (2015)
θ	0.74	Castro *et al.* (2015)
γ_R	0.79	Castro *et al.* (2015)
γ_π	2.43	Castro *et al.* (2015)
γ_Y	0.16	Castro *et al.* (2015)
$\frac{L_{ss}^u}{L_{ss}^{sk}}$	0.509	Average value (2002-2014) PNAD - IBGE
$\frac{W_{ss}^u}{W_{ss}^m}$	0.711	Tannuri-Pianto and Pianto (2016)
$\frac{C_{ss}}{Y_{ss}}$	0.61	IBGE/SCN 2000
$\frac{I_{ss}}{Y_{ss}}$	0.20	IBGE/SCN 2000
$\frac{G_{ss}}{Y_{ss}}$	0.19	IBGE/SCN 2000
$\frac{B_{ss}}{Y_{ss}}$	0.35	Bacen/Not. Imp./F. Pb
Ω_m	0.5	Argentiero and Bollino (2015)

Figure 10.

Figure 10's graphs are especially relevant in that they present key results of this estimation, but they can also serve as tools to detect problems with the said results. First, prior and posterior distributions should not be excessively different from one another. Second, the posterior distributions should follow a normal distribution, or at least not display a shape that is clearly non-normal. Third, the mode (the dotted line) should not be too far away from the mode of the posterior distribution. Overall, it is worth pointing out that the estimates proved to be quite satisfactory.

Table 5: Posterior distribution of the model. Source: authors' calculations.

Parameter	Prior mean	Posterior mean	90% HPD interval	prior	Pstdev
ψ_D	1.000	1.1292	1.1140 1.1460	gamma	0.5000
ψ_F	1.000	0.3071	0.2217 0.4016	gamma	0.5000
ω_{Rf}	0.500	0.4359	0.3600 0.4835	beta	0.2500
ψ^m	1.450	1.4128	1.4067 1.4194	unif	0.0289
ψ^u	1.605	1.6974	1.6949 1.7000	unif	0.0548
τ_{ss}^c	0.159	0.1743	0.1730 0.1755	beta	0.0100
τ_{ss}^l	0.173	0.1761	0.1746 0.1775	beta	0.0100
τ_{ss}^s	0.105	0.1306	0.1273 0.1334	beta	0.0100
τ_{ss}^{corp}	0.300	0.2869	0.2829 0.2924	unif	0.0289
γ_G	0.500	0.5303	0.4824 0.5740	unif	0.2829
γ_{IGm}	0.500	0.3921	0.2952 0.5012	unif	0.2829
γ_{IGu}	0.500	0.9773	0.9635 0.9900	unif	0.2829
γ_T	0.500	0.5598	0.5308 0.5980	unif	0.2829
γ_{τ^c}	0.500	0.0202	0.0100 0.0324	unif	0.2829
$\gamma_{\tau^{corp}}$	0.745	0.5127	0.5000 0.5244	unif	0.1415
γ_{τ^l}	0.500	0.9816	0.9686 0.9900	unif	0.2829
γ_{τ^s}	0.500	0.9322	0.9047 0.9643	unif	0.2829
ϕ_G	-0.500	-0.1347	-0.1874 -0.0744	unif	0.2887
ϕ_{IGm}	-0.500	-0.4496	-0.5037 -0.3896	unif	0.2887
ϕ_{IGu}	-0.500	-0.8391	-0.8637 -0.8105	unif	0.2887
ϕ_T	0.500	0.9528	0.9020 1.0000	unif	0.2887
ϕ_{τ^c}	0.500	0.9900	0.9754 1.0000	unif	0.2887
$\phi_{\tau^{corp}}$	0.150	0.1741	0.1497 0.1980	unif	0.0866
ϕ_{τ^l}	0.500	0.1939	0.1225 0.2648	unif	0.2887
ϕ_{τ^s}	0.500	0.8309	0.7689 0.8899	unif	0.2887
ρ_P	0.5	0.4149	0.4020 0.4286	beta	0.25
ρ_L	0.5	0.7766	0.7545 0.8000	beta	0.25
ρ_{A^m}	0.5	0.8245	0.7701 0.8778	beta	0.25
ρ_{A^u}	0.5	0.8641	0.8250 0.8988	beta	0.25
ρ_{pr}	0.5	0.3330	0.2843 0.3999	beta	0.25
ρ_G	0.5	0.1918	0.1169 0.2599	beta	0.25
ρ_{IGm}	0.5	0.2034	0.1512 0.25	beta	0.25
ρ_{IGu}	0.5	0.3748	0.3225 0.4298	beta	0.25
ρ_T	0.5	0.4166	0.3773 0.4614	beta	0.25
ρ_{τ^c}	0.5	0.5096	0.4826 0.5433	beta	0.25
$\rho_{\tau^{corp}}$	0.5	0.6014	0.5574 0.6409	beta	0.25
ρ_{τ^s}	0.5	0.5383	0.4831 0.5750	beta	0.25
ρ_{τ^l}	0.5	0.3632	0.2856 0.4309	beta	0.25
ρ_m	0.5	0.7867	0.7420 0.8341	beta	0.25
ρ_{φ_m}	0.5	0.6835	0.6080 0.7466	beta	0.25
ρ_{P^F}	0.5	0.1596	0.1090 0.2031	beta	0.25
ρ_{R^F}	0.5	0.4964	0.4228 0.5622	beta	0.25
ρ_{Y^F}	0.5	0.4336	0.3978 0.4671	beta	0.25
ε_P	1.0	5.5792	4.9452 6.2704	invg	Inf
ε_L	1.0	4.1383	3.7837 4.4330	invg	Inf
ε_{A^m}	1.0	0.4437	0.3666 0.5178	invg	Inf
ε_{A^u}	1.0	0.7680	0.5342 0.9816	invg	Inf
ε_{pr}	1.0	2.8992	2.6343 3.1959	invg	Inf
ε_G	1.0	0.2122	0.1176 0.2844	invg	Inf
ε_{IGm}	1.0	0.3056	0.2043 0.4031	invg	Inf
ε_{IGu}	1.0	2.8358	2.4577 3.2938	invg	Inf
ε_T	1.0	1.3583	1.1398 1.5661	invg	Inf
ε_{τ^c}	1.0	0.1778	0.1497 0.2048	invg	Inf
$\varepsilon_{\tau^{corp}}$	1.0	0.5540	0.4664 0.6389	invg	Inf
ε_{τ^s}	1.0	5.5808	5.1794 5.9823	invg	Inf
ε_{τ^l}	1.0	6.4431	5.9503 6.8755	invg	Inf
ε_m	1.0	0.1248	0.1176 0.1334	invg	Inf
ε_{φ_m}	1.0	1.1867	0.3961 2.1075	invg	Inf
ε_{P^F}	1.0	0.1230	0.1176 0.1301	invg	Inf
ε_{R^F}	1.0	0.1213	0.1176 0.1262	invg	Inf
ε_{Y^F}	1.0	0.1208	0.1176 0.1247	invg	Inf
ε_{price}	1.0	0.1258	0.1176 0.1354	invg	Inf
ε_{gov}	1.0	0.2791	0.2084 0.3444	invg	Inf
corr ε_{IGm},ε_{IGu}	0.000	0.9451	0.9038 0.9797	beta	0.3000

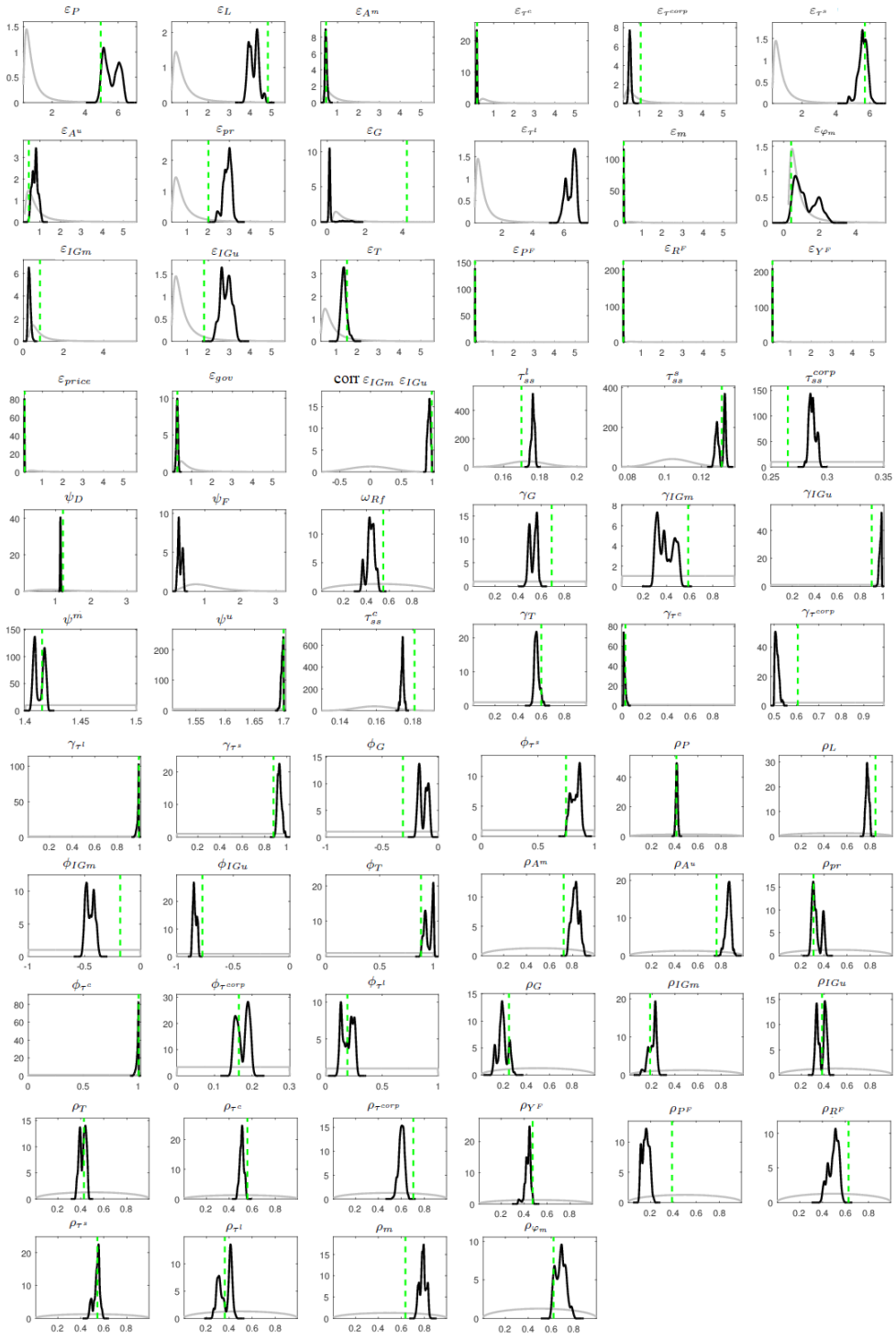

Figure 10: Priors and posteriors. Source: authors' calculations.

Appendix B: Proofs of propositions

Proof. (Proposition 3.1). Assuming that the economy is in its steady state and the fact that households decide the amount of work they are willing to supply through the equations (7) and (17), the informal labor supply in relative terms is:

$$L^u = \left\{ \left(\frac{1}{1-\beta\theta_W} \right) \left(\frac{W^u}{W^m} \right) \left(\frac{\varphi_m}{\varphi_m - 1} \right) \left(\frac{1}{1-\tau^l} \right) \right\}^{\frac{1}{\psi_u}} L^{m\,\frac{\psi_m}{\psi_u}}$$

Let the differentiation of the preceding equation be given by the following equations:

$$\frac{\partial L^u}{\partial \left(\frac{W^u}{W^m} \right)} = \frac{1}{\psi_u} \left\{ \left(\frac{1}{1-\beta\theta_W} \right) \left(\frac{\varphi_m}{\varphi_m - 1} \right) \left(\frac{1}{1-\tau^l} \right) \right\}^{\frac{1}{\psi_u}} L^{m\,\frac{\psi_m}{\psi_u}} \left(\frac{W^u}{W^m} \right)^{\frac{1}{\psi_u}-1}$$

$$\frac{\partial L^u}{\partial \left(\frac{\varphi_m}{\varphi_m - 1} \right)} = \frac{1}{\psi_u} \left\{ \left(\frac{1}{1-\beta\theta_W} \right) \left(\frac{W^u}{W^m} \right) \left(\frac{1}{1-\tau^l} \right) \right\}^{\frac{1}{\psi_u}} L^{m\,\frac{\psi_m}{\psi_u}} \left(\frac{\varphi_m}{\varphi_m - 1} \right)^{\frac{1}{\psi_u}-1}$$

$$\frac{\partial L^u}{\partial \tau^l} = \frac{1}{\psi_u} \left\{ \left(\frac{1}{1-\beta\theta_W} \right) \left(\frac{W^u}{W^m} \right) \left(\frac{\varphi_m}{\varphi_m - 1} \right) \right\}^{\frac{1}{\psi_u}} L^{m\,\frac{\psi_m}{\psi_u}} \left(\frac{1}{1-\tau^l} \right)^{\frac{1}{\psi_u}+1}$$

$$\frac{\partial L^u}{\partial \theta_W} = \frac{\beta}{\psi_u} \left\{ \left(\frac{W^u}{W^m} \right) \left(\frac{\varphi_m}{\varphi_m - 1} \right) \left(\frac{1}{1-\tau^l} \right) \right\}^{\frac{1}{\psi_u}} L^{m\,\frac{\psi_m}{\psi_u}} \left(\frac{1}{1-\beta\theta_W} \right)^{\frac{1}{\psi_u}+1}$$

Since: $\frac{\partial L^u}{\partial \left(\frac{W^u}{W^m} \right)} > 0$; $\frac{\partial L^u}{\partial \left(\frac{\varphi_m}{\varphi_m - 1} \right)} > 0$; $\frac{\partial L^u}{\partial \tau^l} > 0$; and $\frac{\partial L^u}{\partial \theta_W} > 0$.

□

Proof. (Proposition 3.2). Assuming that the economy is in its steady state, and that firms decide the amount of informal labor demanded as described by equations (26) and (32), the relative demand for informal labor is:

$$L^u = \left[(1+\tau^s) \left(\frac{1 - prs\tau^{corp}}{1-\tau^{corp}} \right) \left(\frac{W^m}{W^u} \right) \right] L^m$$

Let the differentiation of the preceding equation be determined by the following equations:

$$\frac{\partial L^u}{\partial \tau^s} = \left[\left(\frac{1 - prs\tau^{corp}}{1-\tau^{corp}} \right) \left(\frac{W^m}{W^u} \right) \right] L^m$$

$$\frac{\partial L^u}{\partial pr} = -\left[(1+\tau^s) \left(\frac{s\tau^{corp}}{1-\tau^{corp}} \right) \left(\frac{W^m}{W_u} \right) \right] L^m$$

$$\frac{\partial L^u}{\partial \tau^{corp}} = \left[\frac{1 - prs}{(1-\tau^{corp})^2} \right] \left[(1+\tau^s) \left(\frac{W^m}{W^u} \right) \right] L^m$$

$$\frac{\partial L^u}{\partial \left(\frac{W^m}{W^u} \right)} = \left[(1+\tau^s) \left(\frac{1 - prs\tau^{corp}}{1-\tau^{corp}} \right) \right] L^m$$

Given that: $\frac{\partial L^u}{\partial \tau^s} > 0$; $\frac{\partial L^u}{\partial pr} < 0$; $\frac{\partial L^u}{\partial \tau^{corp}} > 0$; and $\frac{\partial L^u}{\partial \left(\frac{W^m}{W^u} \right)} > 0$.

□